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Study of Pure Torsion in Open Thin Walled Sections using Finite Element Analysis

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Abstract
Generally, Open Thin Walled (OTW) cross sections are susceptible to torsional instability, due to their low torsional rigidity. Several analytical methods have been proposed to calculate the torsional deformation (warping) along the direction perpendicular to the cross section of such OTW sections when subjected to torsion. In this paper, we have analysed the warping in the OTW sections to validate that in case of pure torsion of such bars having free ends, the warping is uniform throughout the whole length of the bar and any line parallel to centroidal axis can be taken as axis of rotation. The finite element analysis using ABAQUS has been done to analyse the warping of such sections when subjected to pure torsion. This paper also represents the comparative study of warping distribution in OTW sections using the theoretical approach and Finite Element Modeling (FEM).

Keywords- Pure Torsion, Warping, Open Thin Walled Section, ABAQUS, Finite Element Method.

1. Introduction

In some structures, the structural members have to transmit primary torque, or sometimes in combination with bending and axial loads. In such members, the torsion may be secondary undesirable effect which is causing premature failure or deformations. It is quite necessary to analyse and design for such effects like warping due to torsion.

The first approach in the study of torsional analysis was classical torsional theory by Saint-Venant (1853) which suggests that the rate of twist will be constant throughout the member and the warping will be same for all the cross sections when cross sections are free to warp along the direction perpendicular to plane of section. Effects of warping torsion in I- beams has also been presented by Timoshenko (1905) and the existence of warping stresses (not predicted by classical torsional theory) when the shear centre of section is not coinciding with the centroid was later on described by Batch (1909).

Generally, the OTW sections when loaded in compression, fails by twisting even before they reach yield point or Eulerian buckling load. Wagner and Pretschener (1934) have suggested the use of warping displacements for practical calculation of a value of resistance to flexure and torsion.

Researchers proposed many modifications to the classical theory of torsion to overcome the limitation of buckling being restrained to few development orders of rotations and displacements (Guokang, 1997) where in the geometric and equilibrium equations are modified for flexural- torsional buckling analysis.

Using the standard static perturbation technique on one-dimensional domain, certain researchers obtained the flexural torsional buckling loads (Pignataro et al., 2010). This method helps in limiting the numerical errors for parametric solution of buckling equations. This concept is quite applicable to the low torsional rigidity sections like OTW sections.

In case of sections having comparatively large width of flanges, where the deformations due to pre-buckling are omitted, linear buckling analysis using the finite element formulations conservatively predicts the shear deformations as well as position of loads (Erkmen and Attard, 2011).

The transmission of torsional warping at the joints connecting two or more members is difficult to apply during finite element modeling. This complication can be removed by kinematic simulation using Generalized Beam Theory (Basaglia et al., 2012). This theory can be used to analyse global behavior of plane and thin walled space frames in case of in-plane and out-plane deformation of cross-section of members.

The stability analysis of members with varying cross-sections are generally analysed by power-series approximation method (Asgarians et al., 2013). Fourth-order differential equations of OTW beams having variable flexural- torsional stiffness are solved to define the stability of structures.

In this paper the warping at various cross- sections of structural members along the length is calculated using FEM and compared with the results from theoretical approach and the warping pattern along the cross sections has been studied. The classical methods for calculating stability of members are limited up to uniform cross- sections, the present FE approach can be used for any complicated cross- section.

2. System Description

An open thin walled channel section of uniform thickness and homogenous material has been considered for analysis which is characterized by parameters as shown (Figure 1).

3. Methodology

The important background concept necessary to study the effect of pure torsion is described in this section. Timoshenko and Gere (1963) has suggested that if a bar of Open Thin Walled (OTW) cross section is twisted by applying couples at the ends and acting in planes normal to axis of bar, and ends of the bar are free to warp, the bar will experience pure torsion. In such

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case, only shear stresses will be produced at each section of the bar. The analytical approach to calculate warping due to this pure torsion is as described below.

The rate of change of angle of twist (θ) will be equal to the torque (M_t) applied divided by the torsional rigidity (C) of the bar where torsional rigidity will be the product of shearing modulus of elasticity of bar and torsion constant (J).

$$\theta = M_t / GJ \quad (1)$$

The displacements of middle line of cross section of OTW bars in the longitudinal direction of bar will be given by the relation.

$$w = w_0 - \theta \int_0^S r ds \quad (2)$$

Where r is the perpendicular distance of element ds to the shear centre (or distance of axis of rotation from the tangent at element ds in case of curved cross sections) and w_0 is the displacement of the point along longitudinal direction from which distance s is measured (Figure 2).

4. FE Modeling Description

The section assumed in above system description is modeled in ABAQUS using 3-D deformable shell of uniform thickness. Since it is not possible to apply the torque in section without any suitable connection, thus the torque is transferred to the section through a rigid link which is extended from centroidal axis to axis of rotation. The axis of rotation is provided at shear centre as shown in Figure 3. A 4-node doubly curved thin or thick shell, reduced integration, hourglass control, finite membrane strains meshing (S4R) is provided for finite element analysis.

4.1 Force and Boundary Conditions (BCs)

Moment loads are applied at the end nodes of the section along the axis of rotation in such a way that a uniform torsion is generated throughout the section. The section is hinged at the axis of rotation by restricting the translational displacements of all the nodes coinciding with the axis of rotation (Figure 3).

5. Results

The finite element model described above is analysed for warping at various cross section of bar and results are checked as shown in Table 1.

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From Table.1 it can be observed that at any cross section along the length of bar, the warping displacement along z- direction is almost same for pure torsion. The warping behavior of the bar at any cross section along z-direction is shown in Figure 5.

The deformed shape of the OTW section can be visualized from Figure 4 as obtained from FEM visualization module.

5.1 Comparison between Analytical Results and FEM Results

Analytically warping displacement of the bar can be calculated using equations (1) and (2). We have calculated the warping displacement along the cross section at z=500 (section xx) with respect to an initial warping at point A (s=0) for the whole length of middle line of cross section and compared with the results calculated from FE analysis.

In Table 2 the comparison results show a very small difference in warping displacement of middle line of cross section for both analyses which can be graphically seen from Figure 6.

6. Conclusion

A finite element approach has been presented for torsional analysis of OTW sections and it has been used to evaluate the warping displacement along the cross section. Through this work, the method of applying condition of uniform torsion is validated. Also, the shell elements are found to be satisfactory for determining warping displacements in OTW sections.

S	W_0	W_{100}	W_{200}	W_{300}	W_{400}	W_{500}
0	23.636	23.6473	23.6702	23.6702	23.636	23.6473
10	13.4136	13.4273	13.4522	13.4522	13.4136	13.4273
20	3.1913	3.20715	3.23489	3.23489	3.1913	3.20715
30	-7.03055	-7.0132	-6.98178	-6.98178	-7.03055	-7.0132
40	-17.252	-17.2334	-17.1975	-17.1975	-17.252	-17.2334
50	-13.7917	-13.7866	-13.7586	-13.7586	-13.7917	-13.7866
60	-10.3319	-10.3399	-10.3193	-10.3193	-10.3319	-10.3399
70	-6.86655	-6.89307	-6.87958	-6.87958	-6.86655	-6.89307
80	-3.39699	-3.44625	-3.43963	-3.43963	-3.39699	-3.44625
90	-3.58E-14	-1.88E-13	-7.49E-14	7.26E-14	-3.58E-14	-1.88E-13
100	3.39699	3.44625	3.43963	3.43963	3.39699	3.44625
110	6.86655	6.89307	6.87958	6.87958	6.86655	6.89307
120	10.3319	10.3399	10.3193	10.3193	10.3319	10.3399
130	13.7917	13.7866	13.7586	13.7586	13.7917	13.7866
140	17.252	17.2334	17.1975	17.1975	17.252	17.2334
150	7.03055	7.0132	6.98178	6.98178	7.03055	7.0132
160	-3.1913	-3.20715	-3.23489	-3.23489	-3.1913	-3.20715
170	-13.4136	-13.4273	-13.4522	-13.4522	-13.4136	-13.4273
180	-23.636	-23.6473	-23.6702	-23.6702	-23.636	-23.6473

Table 1. Warping along z-direction for various cross sections

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Distance(s)	$U3_s$ (mm)	$W_{FEM}= U3_s-U3_0$ (mm)	$W_{analytical} = W_s-W_0$ (mm)	Difference (W_{FEM} - $W_{analytical}$)
0	23.636	0	0	0
10	13.4136	10.2224	10	-0.2224
20	3.1913	20.4447	20	-0.4447
30	-7.03055	30.66655	30	-0.66655
40	-17.252	40.888	40	-0.888
50	-13.7917	37.4277	37.174	-0.2537
60	-10.3319	33.9679	34.348	0.3801
70	-6.86655	30.50255	31.522	1.01945
80	-3.39699	27.03299	28.696	1.66301
90	-3.58E-14	23.636	25.87	2.234
100	3.39699	20.23901	23.044	2.80499
110	6.86655	16.76945	20.218	3.44855
120	10.3319	13.3041	17.392	4.0879
130	13.7917	9.8443	14.566	4.7217
140	17.252	6.384	11.74	5.356
150	7.03055	16.60545	21.74	5.13455
160	-3.1913	26.8273	31.74	4.9127
170	-13.4136	37.0496	41.74	4.6904
180	-23.636	47.272	51.74	4.468

Table 2. Comparison of warping along z-direction for analytical and FE results at z=500 mm

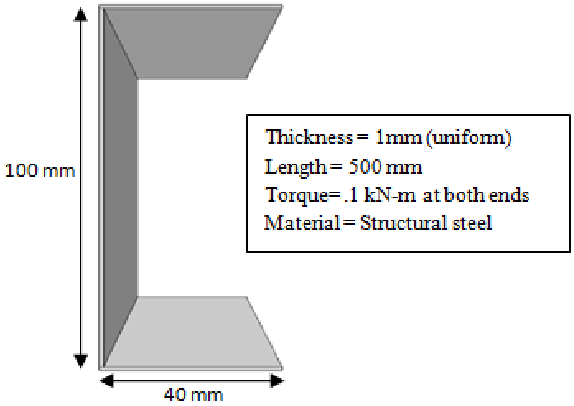

Figure 1. Cross-sectional view of channel

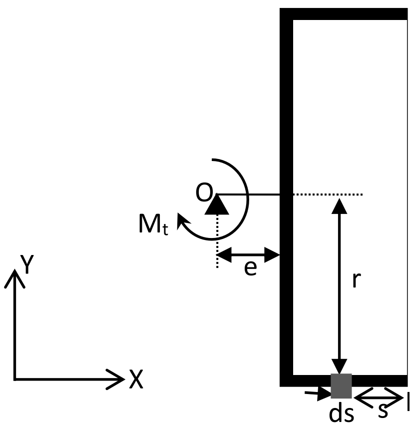

Figure 2. Application of pure torsional load at shear centre

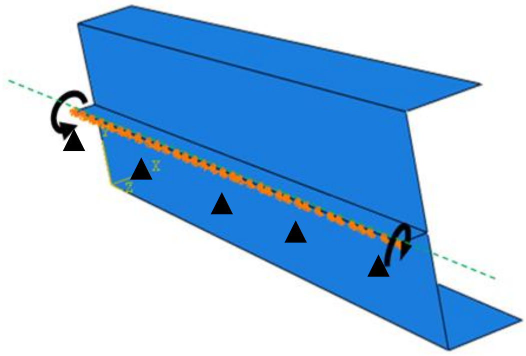

Figure 3. FE Model of open thin walled section for warping analysis

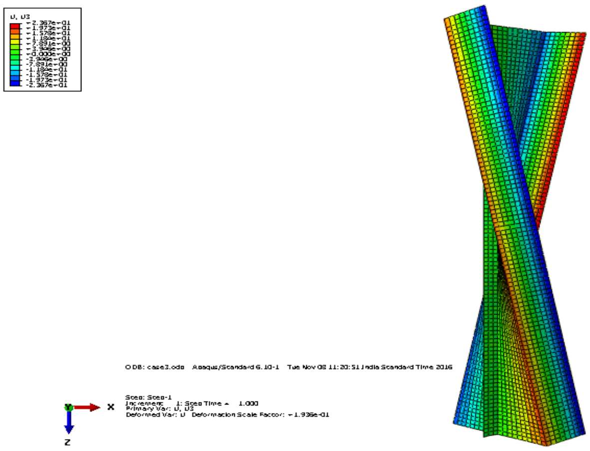

Figure 4. Deformed shape of section under uniform torsion

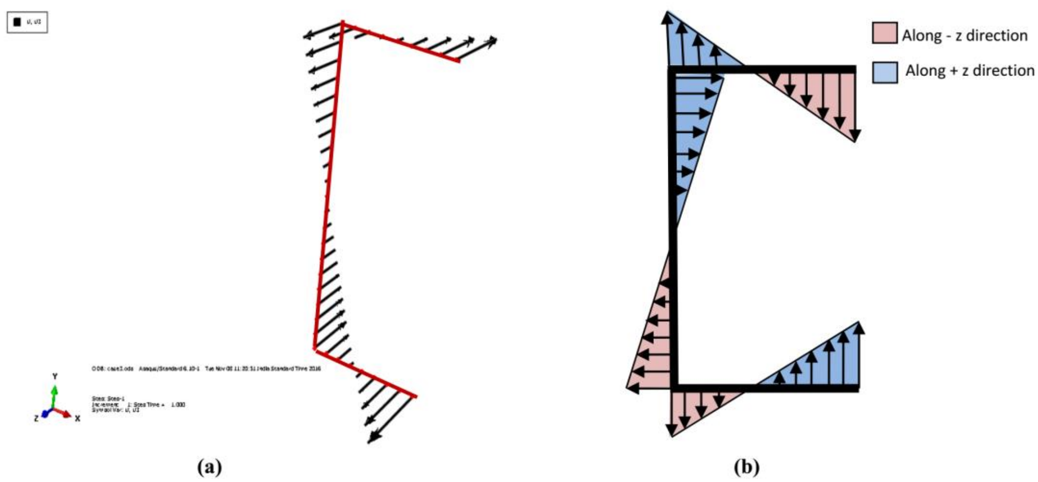

Figure 5. Warping distribution in cross-section (at z=500 mm) of bar along z-direction (a) 3-d view (b) 2-d view

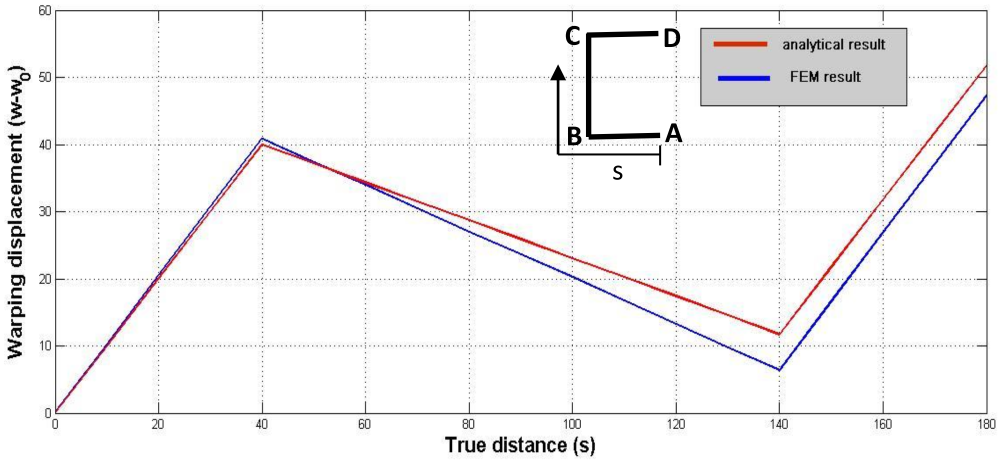

Figure 6. Comparison of warping displacement along z-direction for analytical and FEM results

References

Asgarian, B., Soltani, M., & Mohri, F. (2013). Lateral-torsional buckling of tapered thin-walled beams with arbitrary cross-sections. Thin-Walled Structures, 62, 96-108.

Basaglia, C., Camotim, D., & Silvestre, N. (2012). Torsion warping transmission at thin-walled frame joints: kinematics, modeling and structural response. Journal of Constructional Steel Research, 69(1), 39-53.

Erkmen, R. E., & Attard, M. M. (2011). Lateral–torsional buckling analysis of thin-walled beams including shear and pre-buckling deformation effects. International Journal of Mechanical Sciences, 53(10), 918-925.

Guokang, E. (1997). Modification in the theory on flexural-torsional buckling of structures. Applied Mathematics and Mechanics, 18(10), 975-986.

Pignataro, M., Rizzi, N., Ruta, G., & Varano, V. (2010). The effects of warping constraints on the buckling of thin-walled structures. Journal of Mechanics of Materials and Structures, 4(10), 1711-1727.

Timoshenko, S. P., & Gere, J. M. (1963). Theory of Elastic Stability, 2nd ed., McGraw- Hill International Editions.

Wagner, H., & Pretschener, W. (1934). Torsion and buckling of open sections, technical memorandum. National Advisory Committee for Aeronautics, Washington, 11(784), 1-14.