

Stochastic Flowshop Scheduling Model for Two Machines

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Abstract

In this paper, we have developed a new heuristic algorithm for n jobs two machines ($n \times 2$) flowshop scheduling problem in which processing times is associated with their respective probabilities. The objective of this paper is to find the optimal sequence of jobs to minimize the makespan (total completion times of jobs) and the total mean weighted flow time of jobs. The transportation times of the first machine to second machine are also being considered. Further, jobs are attached to their weight to indicate their relative importance. We also calculated the utilization times of machines. The algorithm is justified by the numerical illustration and Gantt chart is generated to verify the effectiveness of the proposed approaches. The proposed heuristic algorithm is easy to understand and provide an important tool for decision makers.

Keywords- Flowshop Scheduling, Transportation Times, Makespan, Utilization Times of Machine, Weighted Mean Flowtime.

1. Introduction

Scheduling is one of the most mathematical involved and proposed fields in Industrial Engineering and Operations Management. In day to day life, the decision makers are very curious to find the best way to successfully manage the resources in order to produce product of the most efficient way for manufacturing and service industries. Sequencing simply refers to the determination of ORDER in which the jobs are to be processed on various machines. Scheduling refers to the time table that includes the start time and completion times of jobs of machines etc. Resources are usually called machines and tasks are called jobs or operations. The environment of the scheduling problem is called shop. There are different types of shops using in scheduling problems like job shop, flow shop, mixed shop, open shop etc. Here, we deal with flowshop scheduling environment for two machine n jobs and it is shown in Figure 1.

During last 4 decades many researchers work on scheduling and sequencing. Sequencing is the order for n jobs of m machine and scheduling is the process in which set of jobs is sequenced. The idea of flowshop sequencing is given by (Johnson, 1954). The general flow shop scheduling problem is a production problem where a set of n jobs have to be processed with identical flow patterns of m machines. Many researchers studied the different parameters

like Job- Block, Setup Times, Transportation Times, Breakdown of machines, weights of jobs etc. separately. In this paper we study the concept of three parameters (weights of jobs, transportation time and Probabilistic Processing Times) together for two machines flow shop scheduling problem. Furthermore, it is assumed that all jobs processing times are not known in advanced. It means processing times of jobs are stochastic, not deterministic in nature and jobs are also attached with their weights to indicate their relative importance.

In this paper, we combine the three concepts as follows:

- (i) **Weights of the Jobs:** jobs are also attached with their weights to indicate the relative importance of jobs.
- (ii) **Transportation Times:** when the machines are established out different places, on which jobs are to be processed then, transportation time (loading time, moving time and unloading time etc.) has a significant role in production concern.
- (iii) **Probabilistic Processing Time:** In this paper we also assumed that all jobs processing times are not known in advanced. It means processing times of jobs are stochastic, not deterministic in nature.

In our problems we assumed that machines are distantly situated. Therefore, sometime is taken in transferring the job of machine1 to the machine2 in the form of loading time, Moving time and Unloading time of job. So transportation time has remarkable role in production management and if the priority over one job of the other job may be significant due to some urgency or demand of one particular type of job of other. Hence the weights of jobs become important criteria. We developed a new heuristic algorithm to find the optimal sequence of stochastic processing times including transportation times and weights of jobs.

Many researchers introduced the various concepts in different criteria like transportation time, job- block, breakdown of machine, setup time etc. for flowshop scheduling on two or three stages. (Baker, 1974) published a book of scheduling and sequencing. (Tyagi and Chandramouli, 2014) presented the flowshop scheduling model for 2 machines with transportation times and job block. The significant factors on which scheduling problem practically depends are Transportation time, job block, break down effect, Weight of jobs (Relative importance of a job over another job) etc. These conceptions were individually studied by (Mitten, 1959; Smith and Dudek, 1967; Ignall and Schrage, 1965; Brown and Lomnicki, 1966; Heydari, 2003; Chen and Lee, 2008; Gupta, 1975; Maggu and Das, 1977) gives solution algorithm of obtaining optimal sequence for n job 2 machines ($n \times 2$) flowshop scheduling problem when each job involves transportation times. (Singh et al., 2005) gave an algorithm to find the optimal schedule for specially structure flowshop scheduling with setup tim, (Yoshida and Hitomi, 1979; Singh, 1995; Gupta and Singh, 2005; Lomnicki, 1965; Chandermouli, 2005) etc. by considering the various parameters. (Palmer, 1965; Nawaz et al., 1983; Sarin and Lefoka, 1993; Dannenbring, 1977), etc. (Miyazaki and Nishiyama, 1980) associated weight the jobs.

2. Practical Scenario

In real life situation, flowshop scheduling occurs in so many fields such as hospitals, factories, educational intuitions, banking, etc. Two machines flowshop scheduling problem occur to the company like fabricating apparel industries in which two machines are used one is for cutting and second one for sewing. If first machine (cutting) is "M₁" and second machine is "M₂" then processing order for jobs of machines will be "M₁M₂" means first jobs processed on machine "M₁" then operated on Machine "M₂". In our problems we assumed that machines are distantly situated. Therefore, sometime is taken in transferring the job of machine "M₁" to the machine "M₂". In the form of loading time, Moving time and Unloading time of job. So transportation time has remarkable role in production management. In real life situations transportations times should be considering apart from processing time. Sometimes the priority over one job of the other is preferred. It may be because of urgency or demand of its relative importance, the weight of the job becomes important criteria in scheduling problems. Practically, in readymade garments manufacturing plant which has mainly two machines (like cutting and sewing) situated at different places. So transportation time has a significant role in production concern. The time taken by second machine (Sewing) will always be greater that the time taken by first machine (Cutting). Many times different quality of garments is to produce with relative importance. So weights of jobs also become significant. The practical situation may be taken in a production industry; manufacturing industry etc.

3. Problem Description

In this paper we consider n job 2 machines ($n \times 2$) scheduling problem in flowshop environment. We considered two machines "M₁" and "M₂" are situated differently and a set of " n " jobs are to be functioned on these two different situated machines in order "M₁M₂". Jobs processing times are related to their relative probabilities. Weights are also attached their respective jobs. Transportation times $t_{i,1 \rightarrow 2}$ are also taken under consideration for shifting the jobs j_i from machine "M₁" to machine "M₂". Our main aim is to obtain the optimal schedule of all the jobs using Johnson's rule for two machines which minimize the total elapsed times of jobs or makespan, weighted mean flow time. We also calculated the utilization time of both the machines.

3.1 Notations and Parameters Used

- M₁ = First Machine.
- M₂ = Second Machine.
- m_{i1} = Processing time of i^{th} job on M₁ machine.
- m_{i2} = Processing time of i^{th} job on M₂ machine.
- $j_i = i^{\text{th}}$ job (where $i = 1$ to n).
- p_i = Probabilities associated with processing time " m_{i1} " of i^{th} job on M₁ machine.
- q_i = Probabilities associated with processing time " m_{i2} " of i^{th} job on M₂ machine.
- f_i = Flow time of i^{th} job.

- C_{max} = Total completion time of i^{th} job of sequence S_0 on Machines " M_1M_2 ".
- w_i = Weight of i^{th} job.
- w_f = Total weighted flow time.
- \bar{w}_f = Weighted mean flow time.
- U_{M_1} = Utilization times of first machine.
- U_{M_2} = Utilization times of second machine.
- $T_{(\alpha_n)out}^{M_1}$ = Outgoing times of last job on machine M_1 .
- $T_{(\alpha_1)in}^{M_2}$ = Incoming times of the first job on machine M_2 .

3.2 Assumptions

- All the jobs and machine are available at times Zero.
- Fixed set of jobs are considering in this problem. The number of jobs doesn't change. (Static Scheduling problem).
- Processing times of jobs is associated with probabilities $\{\sum_{i=1}^n p_i = 1, 0 \leq p_i \leq 1, \sum_{i=1}^n q_i = 1, 0 \leq q_i \leq 1\}$.
- Release time of jobs $r_j = 0$.
- No machine processes more than one operation at a time.
- No preemption is allowed. Once the jobs are started to operate on the machine, it must be finished before some other functions can start on that machine.
- The machine is assumed to be continuously available and Machine breakdowns or maintenance tasks are not considered.
- Setup times are included in the processing time.

3.3 Performance Measures

In this paper, we dealt with these performance measures as follows:

(i) Completion Time Measures.

- Total completion time, C_{max} = out going time of the last job on the machine M_2 .
- $C_{max} = T_{(\alpha_n)out}^{M_2}$.
- Total weighted flow time, $w_f = \sum_{i=1}^n w_i f_i$ (where flow time, $f_i = C_i - r_i$).
- Weighted mean flow time, $\bar{w}_f = \frac{\sum_{i=1}^n w_i f_i}{\sum_{i=1}^n w_i}$.

(ii) Machines Utilization Measures.

- Utilization time of first machine, U_{M_1} .
- U_{M_1} = Outgoing time of last job on machine M_1 .

$$U_{M_1} = T_{(\alpha_n)out}^{M_1}$$

- Utilization time of second machine, U_{M_2} .

U_{M_2} = Total flow time – incoming time of the first job on machine M_2 .

$$U_{M_2} = T - T_{(\alpha_1)in}^{M_2}$$

3.4 Objective Functions

Objectives Functions

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1. $\min\{ C_{max} = T_{(\alpha_n)out}^{M_2} \}$
2. $\min \{w_f = \sum_{i=1}^n w_i f_i\}$
3. $\min\{\bar{w}_f = \frac{\sum_{i=1}^n w_i f_i}{\sum_{i=1}^n w_i}\}$
4. $\min \{U_{M_1} = T_{(\alpha_n)out}^{M_1}\}$
5. $\min \{U_{M_2} = T - T_{(\alpha_1)in}^{M_2}\}$

3.5 Mathematical Model for $n \times 2$ Flowshop Scheduling

In this paper we consider n job 2 machines ($n \times 2$) flowshop scheduling problem Let a set of " n " jobs $j_i(j_1, j_2, j_3 \dots j_n)$ to be processed on two different machines " M_1 " and " M_2 " in order " M_1M_2 " with processing time " m_{i1} " and " m_{i2} " respectively. The jobs will be processed first on " M_1 " then on " M_2 ". Let " p_i " and " q_i " be the probabilities associated with processing time " m_{i1} " and " m_{i2} " such that,

$$\{\sum_{i=1}^n p_i = 1, 0 \leq p_i \leq 1, \sum_{i=1}^n q_i = 1, 0 \leq q_i \leq 1\}.$$

Let $w_i(w_1, w_2, w_3 \dots w_n)$ weights (importance or priority of jobs) are attached to jobs $j_i(j_1, j_2, j_3 \dots j_n)$. Transportation times $t_{i,1 \rightarrow 2}$ are also taken under consideration for shifting the jobs j_i from machine " M_1 " to machine " M_2 ". The Mathematical model of ($n \times 2$) flowshop scheduling problem in matrix form is represented in Table 1.

4. Proposed Heuristic Algorithm for Two Machine Flow Shop Scheduling

Step 1: First we calculate expected processing time $m'_{i1} = m_{i1} * p_i$ and $m'_{i2} = m_{i2} * p_i$ on machine M_1 and M_2 respectively. We introduced two new fictitious machines (M'_1) and (M'_2) with processing time m'_{i1} and m'_{i2} reduced the problem with new processing time (m'_i).

Step 2: Check the Johnson's condition for two machines flowshop scheduling.

Structural Conditions; or Structural Relationship for Johnson's Algorithm

Consider that either both or one of the complying structural conditions involving the processing time and transportation time of jobs holds:

$$\min(m'_{i1} + t_{i,1 \rightarrow 2}) \geq \max(t_{i,1 \rightarrow 2} + m'_{i2})$$

If it is satisfied, then go to step 3 otherwise uses some other optimization methods like Branch and Bound Technique. In this paper we assume that Johnson's condition for two machines is satisfied so we go to step 3.

Step 3: Now create the two fictitious machines R and S with processing time R_i and S_i respectively.

$$R_i = m'_{i1} + (t_{i,1 \rightarrow 2})$$

$$S_i = m'_{i2} + (t_{i,1 \rightarrow 2}).$$

Step 4: Calculate the weighted flow time R'_i and S'_i for machines R and S respectively. There are two cases to calculate weighted flow time as follows:

- i. If $\min(R_i, S_i) = R_i$
 Then define, $R'_i = \frac{(R_i + w_i)}{w_i}$ and $S'_i = \frac{S_i}{w_i}$.
- ii. If $\min(R_i, S_i) = S_i$
 Then define $S'_i = \frac{(S_i + w_i)}{w_i}$ and $R'_i = \frac{R_i}{w_i}$.

Step 5: Apply the Johnsons' algorithm to obtain the sequence " S_0 " for the modified problem in step 4. Let the sequence is $S_0 = (\alpha_1, \alpha_2, \alpha_3 \dots, \alpha_i \dots \dots, \alpha_n)$ where α_i is the i^{th} position of the job.

Step 6: Prepare In- Out table for the sequences S_0 , which we found in step 5 and compute the total elapsed time (makespan) T , utilization time U_{M_1} and U_{M_2} of machines M_1 and M_2 respectively.

Step 7: Collect all the calculations in tabular form and after analyzing the table, we find the sequence (s) for which $T, w_f, \overline{w_f}$ and U_{M_2} are minimum.

5. Numerical Illustrations

Let 5 jobs are to be processed on two machines with their processing time associated with probabilities, transportation time and weights of jobs are given in Table 2.

5.1. Numerical Solved by Proposed Heuristic Algorithm

As per step 1: Calculate the expected processing time $m'_{i1} = m_{i1} * p_i$ and $m'_{i2} = m_{i2} * p_i$ for machine M_1 and M_2 respectively are as follows in Table 3.

As per step 2: Check the Johnson's condition for two machines,

$$\min(m'_{i1} + t_{i,1 \rightarrow 2}) \geq \max(t_{i,1 \rightarrow 2} + m_{i2})$$

$$\min(m'_{i1} + t_{i,1 \rightarrow 2}) = 12 \text{ and } \max(t_{i,1 \rightarrow 2} + m_{i2}) = 12$$

$$\min(m'_{i1} + t_{i,1 \rightarrow 2}) = \max(t_{i,1 \rightarrow 2} + m_{i2}) = 6$$

Hence, Structural Condition is satisfied go to step 3.

As per step 3: Create two fictitious machines R and S with processing time r_i and s_i respectively. The modified problem as follows in Table 4.

As per step 4: Calculate the weighted flow time r'_i and s'_i for machines R and S respectively, and modify problem as follows in Table 5.

As per step 5: Apply the Johnsons' algorithm to obtain the sequence " S_0 " for the modified problem in step 4.

$$S_0 = (2, 4, 1, 5, 3)$$

As per step 6: Construct In – Out table for the sequence $\{S_0 = (2, 4, 1, 5, 3)\}$ obtained in step 5 and calculate T, w_f, \bar{w}_f and U_{M_2} and it is shown in Table 6.

As per step 7: Collect all the calculations in tabular form and after analyzing the above table obtained in step 7 we get $C_{max}, \bar{w}_f, U_{M_1}$ and U_{M_2} as follows in Table 7.

6. Gantt Chart

Gantt chart is shown in Figure 2, according to In- Out Table 6. Gantt chart is generated to verify the effectiveness of proposed heuristic algorithm. Figure 2 shows the Gantt chart of the optimal sequence $\{S_0(2, 4, 1, 5, 3)\}$ obtained from proposed heuristic algorithm. According to Gantt chart Total Completion Time (Makespan) $C_{max} = 64units$, utilization time of machine1 (M/c1) $U_{M_1} = 54$, utilization time of machine2 (M/c2) $U_{M_2} = 48$ units. With the help of Gantt chart we also calculated the Total Ideal Time of machines. The value of total ideal time of machine is 14 units.

7. Conclusion and Future Research

The main objective of scheduling is to arrive at a position where we will get minimum processing time. In Propose Heuristic Algorithm we find the optimal or near optimal sequence using Johnson's rule for of $(n \times 2)$ flowshop scheduling problem. The purpose to obtain a optimal sequence is to minimize the makespan and mean weighted flow time, with the help of this purposed heuristic algorithm we can also calculated the utilization time of machines. This heuristic algorithm is clarified with the help of numerical illustration. We also generate the Gantt chart to verify the effectiveness of the proposed heuristic algorithm.

Future research should address problems with different shop environments, including Job Shop, Open Shop, Mixed Shop, Flexible flowshop, parallel machines flow shop etc. For future research different parameters can also be used such as job block, setup times, breakdown of machines, rental cost of machines etc for two or three machines. Exact Algorithm like Branch and Bound is also used for future research. Heuristic (Palmer, CDS,

NEH etc) and Metaheuristic (Genetic Algorithm, Ant Colony Optimization etc.) approach is also used for solving this type of flowshop scheduling problem.

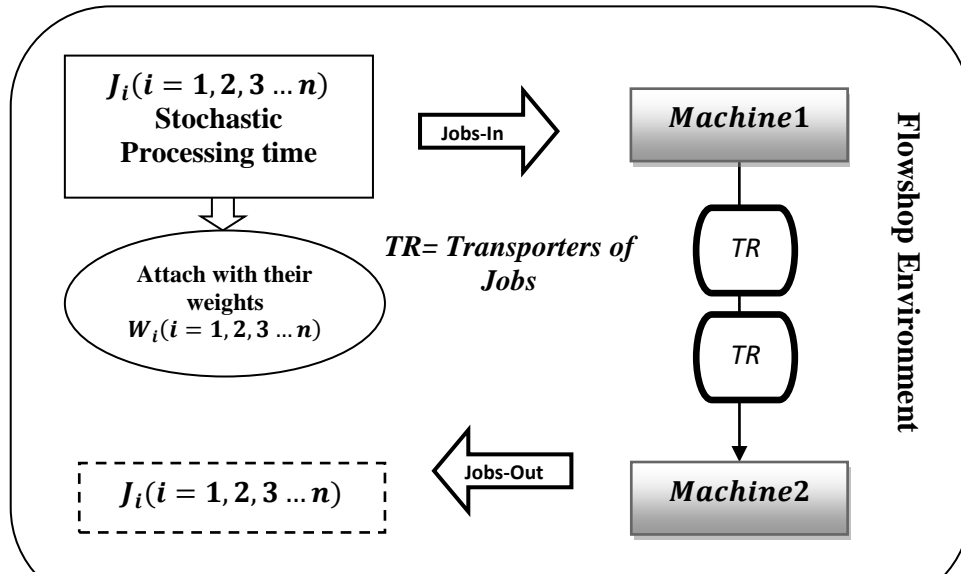


Figure 1. Two stage n jobs flowshop scheduling environment with TR

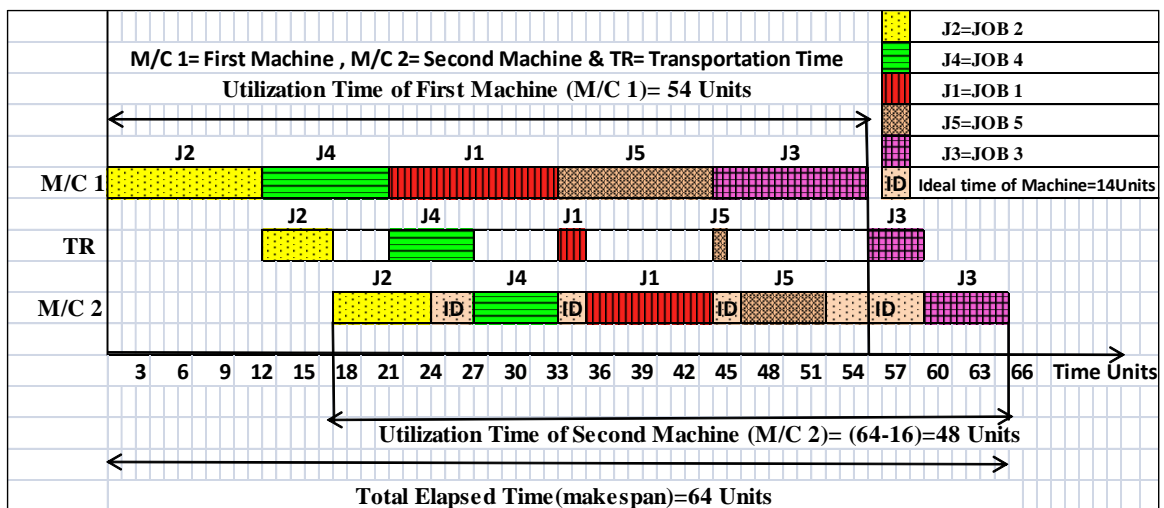


Figure 2. Gantt chart for the optimal sequence $\{S_0 = (2, 4, 1, 5, 3)\}$

Jobs	Machine M_1		Transportation Time	Machine M_2		Weight of jobs
(i)	(m_{i1})	(p_i)	$(t_{i,1 \rightarrow 2})$	(m_{i2})	(q_i)	(w_i)
1	m_{11}	p_1	$t_{1,1 \rightarrow 2}$	m_{12}	q_1	w_1
2	m_{21}	p_2	$t_{2,1 \rightarrow 2}$	m_{22}	q_2	w_2
3	m_{31}	p_3	$t_{3,1 \rightarrow 2}$	m_{32}	q	w_3
⋮	⋮	⋮	⋮	⋮	⋮	⋮
n	m_{n1}	p_n	$t_{n,1 \rightarrow 2}$	m_{n2}	q_n	w_n

Table 1. Mathematical model of $(n \times 2)$ flowshop scheduling problem in matrix form

Jobs	Machine M_1		Transportation Time	Machine M_2		Weight of jobs
(i)	(m_{i1})	(p_i)	$(t_{i,1 \rightarrow 2})$	(m_{i2})	(q_i)	(w_i)
1	120	.10	2	60	.15	1
2	44	.25	5	17.5	.40	6
3	50	.20	4	120	.05	2
4	60	.15	6	20	.30	4
5	40	.30	1	60	.10	1

Table 2. (5×2) flowshop scheduling problem in matrix form

Jobs	Machine M_1	Transportation Time	Machine M_2	Weight of jobs
(i)	(m'_{i1})	$(t_{i,1 \rightarrow 2})$	(m'_{i2})	(w_i)
1	12	2	9	1
2	11	5	7	6
3	10	4	6	2
4	9	6	6	4
5	12	1	6	1

Table 3. Calculate the expected processing time

Jobs	Machine (R)	Machine(S)	Weight of jobs
(i)	$r_i = m'_{i1} + (t_{i,1 \rightarrow 2})$	$s_i = m'_{i2} + (t_{i,1 \rightarrow 2})$	(w_i)
1	14	11	1
2	16	12	6
3	14	10	2
4	15	12	4
5	13	7	1

Table 4. Create two fictitious machines R and S

Jobs	Machine (R)	Machine (S)
(i)	r'_i	s'_i
1	14	12
2	2.7	3
3	7	6
4	3.8	4
5	13	8

Table 5. Calculate the weighted flow time

In-Out table for sequence, $S_0(2, 4, 1, 5, 3)$							
Jobs	Machine M_1		Transportation Time	Machine M_2		Weight of jobs	Weighted flow time
(i)	In	Out	$(t_{i,1-2})$	In	Out	(w_i)	$(w_i f_i)$
2	0	11	5	16	23	6	$(23-0) \times 6 = 138$
4	11	20	6	26	32	4	$(32-11) \times 4 = 84$
1	20	32	2	34	43	1	$(43-20) \times 1 = 23$
5	32	44	1	45	51	1	$(51-32) \times 1 = 19$
3	44	54	4	58	64	2	$(64-44) \times 2 = 40$
	$U_{M_1} = 54$		$U_{M_2} = 64 - 16 = 48, T = 64$			$\sum_{i=1}^5 w_i = 14$	$\sum_{i=1}^5 w_i f_i = 304$

Table 6. Construct In – Out table for the sequence " S_0 "

Optimal Sequence	C_{max}	$\bar{w}_f = \frac{\sum_{i=1}^n w_i f_i}{\sum_{i=1}^n w_i}$	U_{M_1}	U_{M_2}
$S_0 = (2, 4, 1, 5, 3)$	64 units	$304/14 = 21.71$ units	54 units	48 units

Table 7. Result of the objective function in tabular form

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