

An EOQ Model with Stock and Time Dependent Consumption Rate with Shortages under Inflationary Environment

Yogendra Kumar Rajoria*, Seema Saini, Anand Chauhan

Department of Mathematics Graphic Era University, Dehradun, India *Corresponding author: yogendrarajo@mail.com

Abstract

In this paper, we developed an Economic order quantity model for decaying goods with stock and time reliant demand rate under inflationary environment and time discounting over a fixed time horizon. Shortages are also permitted and are partially backordered. The results are discussed with a numerical example and particular cases are also discussed briefly.

Keywords- Inflation, Time value of money, Time horizon.

1. Introduction

In traditional EOQ models, the demand rate of consumed items is unspecified to be either steady or dependent on time, but used for positive kinds of inventory, e.g. buyer goods the consumption rate may be controlled by the stock level, i.e. consumption or expenditure rate possibly will ascend or down according as the on-hand stock level. It is generally viewed that bulky stack of supplies on self in superstore will lead the purchaser to procure more and initiate the higher demand. These facts create a center of attention to many promotion investigators to extend inventory model associated to stock level. Inflation is the percentage change in the value of the price-level. Gupta and Vrat (1986) gave an idea to develop an inventory model to reduce the price with the supposition that stock-dependent expenditure price is a reason of the primary stock level. Padmanabhan and Vrat (1988) considered stockdependent consumption rate as a function of inventory level at any instant of time. Urban (1995) developed a model and assumed that the primary stock and the immediate stock are dependent on demand rate which more realistic. Ray and Chaudhuri (1997) formulated an inventory model under consideration of following assumption (a) time value of money (b) shortages (c) stock-dependent demand rate. Datta and Paul (2001) discussed the inventory system in which demand rate depends on price and stock-dependent. Balkhi and Benkherouf (2004) presented an inventory model for deteriorating items with stock-dependent and time varying demand rates over a finite planning horizon. Shah and Soni (2008) proposed an inventory model for stock-dependent demand rate under the progressive payment scheme. Goyal and Chang (2009) developed an inventory model with stock-dependent demand rate in which they assumed that the retailer gets the delivery of the item and some of the items are displayed in the shop while the rest of the items are kept in the backroom/warehouse. Yang et al. (2010) developed the best possible replenishment strategy under price rises for fading



items with stock-dependent expenditure rate and partial backordering shortages. Patra and Ratha (2012) developed a replenishment policy for decaying items with a stock dependent consumption market under shortage and price rises. Sarkar and Sarkar (2013) formulated a model with time and stock-dependent demand for decaying products with allowable shortages. Singh and Sharma (2014) studied an optimal trade-credit policy for perishable items deeming imperfect production and stock dependent demand. Chauhan and Singh (2014) discussed a best possible replacement and ordering strategy for time dependent demand and deterioration with low-cost cash flow investigation. Chauhan et al. (2016) proposed inventory model with a best possible replacement strategy for consumable substances taking account of time value of money.

This article can be as a growth of prior inventory models by included the individual property of deterioration, the inflation and the time value of money on the best possible replenishment approach over a predetermined time horizon, where the expenditure rate is presumed to be dependent on existing stock-level as well as dependent on moment in time and the shortages are partially backlogged.

2. Notations and Assumptions

The following notations are used to develop the model:

- *r* : Money off rate, standing for the time value of money;
- α : Positive constant parameter;
- *a* : demand parameter, a > 0;
- b : demand parameter, b > 0;
- β : Stock-dependent parameter;
- γ : Time dependent parameter;
- δ : Backlogging parameter, $0 < \delta < 1$;
- *i* : Inflation rate;
- I(t) : Level of inventory at any time t;
- R : r-i, standing for the net money off rate of price rises is constant;
- *H* : Time horizon;
- *T* : The replenishment cycle;
- *m* : The number of refill in the time horizon n = H/T;
- T_j : The whole time that is elapsed up to and plus the jth refill cycle (j=1,2,...,n) where $T_0 = 0, T_1 = T$, and $T_n = H$;
- *t_j* : The point in time at which the inventory level in the jth refill cycle skip down to zero (j=1,2,...,n);
- T_{j-t_j} : Time phase when shortages occur (j=1,2,...,n);
- Q : The 2nd, 3rd....nth refill lot size;
- *I_m* : Highest level of inventory;
- *A* : Ordering cost /refill;
- C : Item cost / unit time;



- C_h : Holding cost / unit/ unit time;
- C_s : Shortage cost /unit /unit time;
- *C*_o : Opportunity cost / unit / unit time due to misplaced deal;
- TRC : Present worth of total cost for the duration of initial refill cycle;
- TC(m, k): Total cost of inventory more than fixed time inventory;

The following assumptions are used to develop the model.

- (i) The expenditure rate D(t) at any point in time t is supposed to be $\alpha + \beta I(t) + \gamma t$, where α is a positive constant, β is the stock-dependent demand rate parameter, $0 \le \beta$, $\gamma \le 1$, as well as I(t) is the level of inventory at any moment of time t.
- (ii) The rate of replenishment is not finite and nil lead time.
- (iii) The time horizon is fixed.
- (iv) Shortages are allowed which are backlogged at the rate of $e^{-\delta t}$ where $0 < \delta < 1$ and *t* is the passing time for next refill.
- (v) Deterioration rate is a + b t; a, b > 0.
- (vi) Product dealings are pursued by instantaneous hard cash flow.

3. Mathematical Model

Assume the time horizon *H* is segregated into *n* one and the same intervals of length T = H/n. as shown in Figure 1. Therefore the reorder times over the time horizon *H* are $T_j = jT$ (j=1, 2,....n). The time for which there is rejection of shortage in every gap [jT, (j+1)T] be a portion of the preparation period *T* and is identical to kT, (0 <k<1). Shortages take place at instance $t_j = (k+j-1)T$, (j=1,2,....n) and are collected until time t=jT (j=1,2,....n)and shortages are backlogged exponential.

Maximum quantity of stock items is replenished at $T_0=0$ during the first replenishment cycle. The inventory level declines in time interval $[0, t_1]$ due to stock-dependent demand and deterioration. It reached zero at $t = t_1$ and shortages takes place for the duration of the time interval $[t_1, T]$ and accumulated until $t = T_1$.

Differential equation based on stock level are given by

$$I'(t) + (a+b t) I(t) = -[\alpha + \beta I(t) + \gamma t] \text{ with } I(t_1) = 0, 0 \le t \le t_1$$
(1)

$$I'(t) = -(\alpha + \gamma t) e^{-\delta t}, t_1 \le t \le T$$
(2)

The respective results of differential equations are

$$I(t) = \alpha \exp\left(-(a+\beta)t - \frac{bt^2}{2}\right) \int_t^{t_1} (\alpha + \gamma x) \exp\left((a+\beta)x + \frac{bx^2}{2}\right) dx, 0 \le t \le t_1$$
(3)

and

$$I(t) = -\frac{1}{\delta^2} \Big[(\delta \alpha + \gamma) \Big(e^{-\delta t_1} - e^{-\delta t} \Big) + \delta \gamma \Big(t_1 e^{-\delta t_1} - t e^{-\delta t} \Big) \Big], t_1 \le t \le T$$

$$\tag{4}$$



Journal of Graphic Era University Vol. 4, Issue 2, 103-112, 2016 ISSN: 0975-1416 (Print)

The maximum inventory level during first replenishment cycle is

$$I(0) = I_m = \int_0^{t_1} (\alpha + \gamma x) e^{\left((a+\beta)t + \frac{bt^2}{2}\right)} dt$$
(5)

and the upper limit of shortage amount for the duration of the first replenishment, cycle which is backordered

$$I_{b} = \frac{1}{\delta^{2}} \left[(\delta \alpha + \gamma) \left(e^{-\delta t_{1}} - e^{-\delta T} \right) + \delta \gamma \left(t_{1} e^{-\delta t_{1}} - T e^{-\delta T} \right) \right]$$
(6)

The present worth of the ordering cost for the period of initial replenishment cycle is A, as the replenishment is finished at the beginning of every one cycle.

And the present worth of the holding cost of inventory for the duration of initial replenishment cycle is

$$H. \ C. = C_h \int_0^{t_1} I(t) e^{-Rt} dt$$

= $C_h \left[\int_0^{t_1} \exp\left(-(a+\beta)t - \frac{bt^2}{2}\right) \left\{ \int_t^{t_1} (\alpha + \gamma x) \exp\left((a+\beta)x + \frac{bx^2}{2}\right) dx \right\} e^{-Rt} dt \right]$ (7)

The present worth of the shortage cost for the duration of initial replenishment cycle is

$$SC = C_{s} \int_{t_{1}}^{T} \frac{1}{\delta^{2}} \Big[(\delta\alpha + \gamma) \Big(e^{-\delta t_{1}} - e^{-\delta t} \Big) + \delta\gamma \Big(t_{1} e^{-\delta t_{1}} - t e^{-\delta t} \Big) \Big] e^{-Rt} dt$$

$$= \frac{C_{s}}{\delta^{2}} \Big[\frac{\delta\alpha + \gamma}{R(\delta + R)} \Big\{ \delta e^{-(\delta + R)t_{1}} - \Big\{ (\delta + R) e^{-\delta t_{1}} - \operatorname{Re}^{-\delta T} \Big\} e^{-RT} \Big\}$$

$$+ \frac{\delta\gamma}{R(\delta + R)^{2}} \Big\{ \Big(\delta(\delta + R)t_{1} - R \Big) e^{-(\delta + R)t_{1}} - \Big\{ (\delta + R)^{2} t_{1} e^{-\delta t_{1}} - R(\delta + R) T e^{-\delta T} - R e^{-\delta T} \Big\} e^{-RT} \Big\} \Big]$$

$$(8)$$

The present worth of material cost for the duration of the first cycle [0, t₁] $C_{p} = CI_{m} + \frac{Ce^{-RT}}{\delta^{2}} \Big[(\delta\alpha + \gamma) \Big(e^{-\delta t_{1}} - e^{-\delta T} \Big) + \delta\gamma \Big(t_{1}e^{-\delta t_{1}} - Te^{-\delta T} \Big) \Big]$ (9)

Opportunity cost payable to lost sale

$$O. C. = C_o \int_{t_1}^{T} (\alpha + \gamma t) (1 - e^{-\delta t}) e^{-Rt} dt$$

$$= C_o \left[\frac{e^{-Rt_1}}{R^2} \{ \gamma + R(\alpha + \gamma t_1) \} - \frac{e^{-(\delta + R)t_1}}{(\delta + R)^2} \{ \gamma + (\delta + R)(\alpha + \gamma t_1) \} - \frac{e^{-RT}}{R^2} \{ \gamma + R(\alpha + \gamma T) \} + \frac{e^{-(\delta + R)T}}{(\delta + R)^2} \{ \gamma + (\delta + R)(\alpha + \gamma T) \} \right]$$
(10)





Therefore, the present worth of total cost of the organization for the duration of the first replenishment cycle is

$$TRC = A + H.C. + S.C. + C_p + O.C.$$
(11)

The present worth of entirety cost of the organization over a fixed time horizon H is

$$TC(m, k) = \sum_{j=1}^{m-1} TRC \ e^{-R \ jT} - Ae^{-RH} = TRC \left(\frac{1 - e^{-RH}}{1 - e^{-RH}}\right) - Ae^{-RH}$$
(12)

The present worth of entirety cost TC (n, k) is a function of two variables n and k, where n is a discrete variable and k is a continuous variable. For a known value of n, the essential condition for TC (m, k) to be minimized is $\frac{dTC(m, k)}{dk} = 0$ which gives

$$\frac{C_{h}H}{m}\left(\alpha + \frac{\gamma kH}{m}\right)\exp\left[\left(a+\beta\right)\frac{kH}{m} + \frac{b}{2}\left(\frac{kH}{m}\right)^{2}\right]\int_{0}^{\frac{kH}{m}}\exp\left[-\left(a+\beta\right)t - \frac{bt^{2}}{2}\right]e^{-Rt}dt$$

$$+\frac{C_{s}H}{\delta Rm}\left[\left(\delta\alpha + \gamma\right)\left\{\exp\left[\frac{\left(\delta k+R\right)H}{m}\right] - \exp\left[\frac{\left(\delta+R\right)kH}{m}\right]\right\} - \gamma\left[1 - \frac{\delta kH}{m}\right]\right]$$

$$\times\exp\left(-\frac{\left(\delta k+R\right)H}{m}\right) + \frac{\gamma}{\left(\delta+R\right)}\left\{\left\{\delta - \left(\frac{\delta\left(\delta+R\right)kH}{m} - R\right)\right\}\exp\left(-\frac{\left(\delta+R\right)kH}{m}\right)\right\}\right]$$

$$+\frac{CH}{m}\left(\alpha + \frac{\gamma kH}{m}\right)\left[\exp\left(\left(a+\beta\right)\frac{kH}{m} + \frac{b}{2}\left(\frac{kH}{m}\right)^{2}\right) - \exp\left(\frac{\left(\delta k+R\right)H}{m}\right)\right]$$

$$\frac{C_{o}H}{m}\left(\alpha + \frac{\gamma kH}{m}\right)\left[\exp\left(-\frac{\left(\delta+R\right)kH}{m}\right) - \exp\left(-\frac{RkH}{m}\right)\right] = 0$$
(13)

Provided the condition $\frac{d^2 T C(m,k)}{dk^2} > 0$ is satisfied (14)

Let (m^*, k^*) denote the best possible answer of *TC* (m, k) and (m, k(m)) indicate the best possible result to *TC*(m, k) when *m* is known. If \tilde{m} is the least integer such that *TC* $(\tilde{m}, k(\tilde{m}))$ is a lesser amount of *TC* $(\tilde{m}, k(\tilde{m}))$ in the time $\tilde{m}+1 < \tilde{m} < \tilde{m}+10$. Subsequently we obtain $(\tilde{m}, k(\tilde{m}))$ the same as the best possible result to *TC*(m, k(m)). Therefore $(\tilde{m}, k(\tilde{m})) = (m^*, k^*)$. Utilizing the best possible course of action explained above, we can get the highest inventory level and best possible order quantity to be

$$I_m = \int_0^{\frac{k^2 H}{m^*}} (\alpha + \gamma x) \exp\left((a + \beta)x + \frac{bx^2}{2}\right) dx$$
(15)

and

$$Q^{*} = \int_{0}^{\frac{k^{*}H}{m^{*}}} (\alpha + \gamma x) \exp\left(_{(a+\beta)x+\frac{bx^{2}}{2}}\right) dx$$

+
$$\frac{1}{\delta^{2}} \left[(\delta\alpha + \gamma) \left(e^{-\delta k^{*}H/m^{*}} - e^{-\delta H/m^{*}} \right) + \delta\gamma \left(\frac{k^{*}H}{m^{*}} e^{-\delta k^{*}H/m^{*}} - \frac{H}{m^{*}} e^{-\delta H/m^{*}} \right) \right]$$
(16)

and behavior of total cost TC (m, k) with respect to time t shown in Figure 2.



4. Numerical Analysis

Input data The proposed model in this paper is illustrated by an example with $\alpha = 500$, $\beta = 0.05$, $\gamma = 0.5$, a = 0.1, b = 0.01, A = 300, C= 12/unit, C_h= 2/unit/year, C_s=10/unit/year, C_o= 15/unit/year, R = 0.2, $\delta = 0.5$, H = 10 year.

Output Data Appling the result process as explained above, the results are put into a Table 1. We noticed that the number of replenishments m = 18, the entirety cost TC(m, k) happen to least amount after seeing in Table 1 and Figure 3. Therefore the best possible values of m and k are $m^* = 18$ and $k^* = 0.551402$, correspondingly, and the least amount of entirety cost $TC(m^*, k^*) = 30861.20$. We after that contain $T^* = H/m^* = 0.5556$, $_{r_1^*} = k^*T^* = 0.306334$, and $Q^* = 257.56$.

5. Sensitivity Analysis

We now study the effects of changes in the values of the system parameters on the different costs and the best possible entirety profit the consequences are summarized in Table 1 and Table 2.

6. Observations

From the Table 1 and Table 2, we observe following conclusion:

- (i) The one hundredth changes in the optimal cost is approximately the same for equally positive and negative changes of all the parameters except C, C_s , C_o , and R.
- (ii) It is seen that the model is more sensitive to a negative change than an equal positive change in either of the parameters C, C_s , C_o , and R.
- (iii) The optimal cost increases (decreases) and decreases (increases) with the increase (decrease) and decrease (increase) in the values of all the parameters *A*, *a*, α , β , *C*_h, and δ . But this trend is reversed for the parameter *R*.
- (iv) It is observed that the parameters C, C_s , C_o , and R. are highly sensitive to negative changes than an equal positive change.
- (v) It can be seen that the model is highly sensitive to changes in α , C, and R, and moderately sensitive to changes in A, C_s , C_o . It has low sensitivity to a, β , C_h , δ , and insensitive to b.
- (vi) From Table 2, we obtain that the effect of R on TC is moderately important for 50% below evaluation of R. It involves with the intention of the result of inflation and time value of cash on the present worth of total cost is very important.
- (vii) Combined effect of various realistic situations on total cost of the system with our model which is shown in Table 3.

7. Conclusion

This study employs properly the traditional inventory lot-size model to allow for price raises i.e., inflation and time value of money. Due to high price rises and subsequent fast refuse in the purchasing power of money, particularly in the growing countries, the financial





circumstances have been changed and so it is not possible to pay no awareness of the effect of the inflation and time value of money. The complexity of inventory systems under an inflationary environment has established interest in current years. This model will be beneficial because, the effect of R (net discount rate of inflation) on total coat C is quite significant for 50% over or under estimation of R. It implies that the effect of inflation and time value of money on present value of total cost is highly significant. The model is very useful in the retail business. It can be used for electronic components, fashionable clothes, domestic goods and other products. The future model includes numerous previous models as particular situation. The model can extend by considering pricing and quality strategies. Also, we could extend the deterministic model into a stochastic model.



Figure 1. The graphical representation of inventory cycle











Figure 3. Variation of total cost TC(m, k) w.r.t. replenishment

М	<i>k</i> (<i>m</i>)	Т	t_1	Q	TC(m, k)
2	0.277281	5.0000	1.386400	1192.71	38371.20
3	0.344346	3.3333	1.147820	1003.30	36445.30
4	0.389999	2.5000	0.974998	854.27	35002.40
5	0.422933	2.0000	0.845866	739.32	33944.10
6	0.447738	1.6667	0.746230	649.61	33160.20
7	0.467054	1.4286	0.667220	578.29	32571.50
8	0.482498	1.2500	0.603123	520.50	32123.90
9	0.495166	1.1111	0.550129	472.88	31780.80
10	0.505611	1.0000	0.505611	433.04	31516.80
11	0.514472	0.9091	0.467702	399.25	31314.10
12	0.522049	0.8333	0435041	370.26	31159.40
13	0.528602	0.7692	0.406617	345.13	31043.40
14	0.534323	0.7143	0.381659	323.15	30958.60
15	0.539359	0.6667	0.359573	303.78	30899.50
16	0.543827	0.6250	0.339892	286.57	30861.80
17	0.547817	0.5882	0.322245	271.18	30842.10
18^{*}	0.551402^{*}	0.5556^{*}	0.306334*	257.35*	30837.70^{*}
19	0.554639	0.5263	0.291915	244.56	30846.30
20	0.557577	0.5000	0.278789	233.51	30866.10
21	0.560255	0.4762	0.266788	223.16	30895.70
22	0.562706	0.4545	0.255775	213.68	30933.80
23	0.564958	0.4348	0.245634	204.97	30979.40
24	0567034	0.4167	0.236264	196.94	31031.60
25	0.568954	0.4000	0.227582	189.52	31089.70
26	0.570735	0.3846	0.219513	182.63	31152.90
27	0.572391	0.3704	0.211997	176.22	31226.40
28	0.573935	0.3571	0.204977	170.24	31293.00

Table 1. Sensitivity analysis with respect to replenishment





Table 2. Sensitivity analyses with respect to different parameters

m^*	k^*	t_1^*	T^*	Order quantity	Present worth total cost
				Q^* (units)	$TC^*(m, k)$
18	0.551402	0.306334	0.5556	257.56	30861.20
16	0.750363	0.468977	0.6250	302.74	69669.60
18	0.552326	0.306848	0.5556	257.58	30858.70
17	0.579338	0.340787	0.5882	271.81	30732.70
18	0.551402	0.306334	0.5556	257.29	30829.90
	m* 18 16 18 17 18	m^* k^* 18 0.551402 16 0.750363 18 0.552326 17 0.579338 18 0.551402	m^* k^* t_1^* 18 0.551402 0.306334 16 0.750363 0.468977 18 0.552326 0.306848 17 0.579338 0.340787 18 0.551402 0.306334	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	m^* k^* t_1^* T^* Order quantity Q^* (units)180.5514020.3063340.5556257.56160.7503630.4689770.6250302.74180.5523260.3068480.5556257.58170.5793380.3407870.5882271.81180.5514020.3063340.5556257.29

Table 3. Comparison of the effect for the above particular situation



References

Balkhi, Z. T., & Benkherouf, L. (2004). On an inventory model for deteriorating items with stock dependent and time-varying demand rates. Computers & Operations Research, 31(2), 223-240.

Sharma, V., Chauhan, A., & Kumar, M. (2016). EOQ models with optimal replenishment policy for perishable items taking account of time value of money. Indian Journal of Science and Technology, 9(25), 1-19.

Chauhan, A., & Singh, A. P. (2014). Optimal replenishment and ordering policy for time dependent demand and deterioration with discounted cash flow analysis. International Journal of Mathematics in Operational Research, 6(4), 407-436.

Datta, T. K., & Paul. A. K. (2001). An inventory system with stock-dependent, price-sensitive demand rate. Production Planning and Control, 12(1), 13-20.

Gupta, R., & Vrat, P. (1986). Inventory model for stock-dependent consumption rate. Opsearch, 23, 19-24.

Goyal, S. K. & Chang. C. T. (2009). Optimal ordering and transfer policy for an inventory with stock dependent demand. European Journal of Operational Research, 196, 177–185.

Padmanabhan, G., & Vrat, P. (1988). Inventory models for perishable items under stock dependent consumption rate. In XXIst Annual ORSI Convention, 1998, Trivandrum, India.

Patra, S. K., & Ratha, P. C. (2012). An inventory replenishment policy for deteriorating items under inflation in a stock dependent consumption market with shortage. International Journal of Tran disciplinary Research, 6 (1), 1-23.

Ray, J. & Chaudhuri, K. S. (1997). An EOQ model with stock-dependent demand shortage inflation and time discounting. International Journal of Production Economics, 53, 171–180.

Shah. N. H., & Soni. H., (2008). Optimal ordering policy for stock-dependent demand under progressive payment scheme. European Journal of Operational Research, 184, 91–100.

Sarkar, B., & Sarkar, S. (2013). An improved inventory model with partial backlogging, time varying deterioration and stock-dependent demand. Economic Modelling, 30, 924-932.

Singh, S. R., & Sharma, S. (2014). Optimal trade-credit policy for perishable items deeming imperfect production and stock dependent demand. International Journal of Industrial Engineering Computations, 5(1), 151-168.

Urban, T. L. (1995). Inventory models with the demand rate dependent on stock and shortage levels. International Journal of Production Economics, 40(1), 2–28.

Yang, H. L., Teng, J.T., & Chern, M. S. (2010). An inventory model under inflation for deteriorating items with stock-dependent consumption rate and partial backlogging shortages. International Journal of Production Economics, 123, 8-19.