T Violation in Four Flavour Neutrino Oscillation in Planck Scale

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Received 27 August 2022; Accepted 03 September 2022;
Publication 06 October 2022

Abstract

The Planck scale effects have been studied in the four flavour, we discuss the Planck scale effects in the four flavour neutrino sector on the asymmetry between T-conjugate oscillation probabilities. $\Delta P_T = P(\nu_\alpha \rightarrow \nu_\beta) - P(\nu_\beta \rightarrow \nu_\alpha)$, for four flavor framework. In this paper, we also discuss some aspect of T violation effects in four flavor neutrino oscillation above the GUT scale.

Keywords: T violation, four flavor mixing.

1 Introduction

The deficit of neutrinos flux suggested the tiny mass of neutrins. Neutrinos are not massless and mixing in the lepton sector, this indicates that there is T/CP violation. Three flavour neutrino oscillation probability in general depends on six parameter three mixing angles $\theta_{13}, \theta_{12}, \theta_{23}$, one CP violating phase δ and two independent mass square difference Δ_{21} and Δ_{31}, The current best fit value of neutrino mixing angle and mass square difference from the

Journal of Graphic Era University, Vol. 10_2, 217–228.
doi: 10.13052/jgeu0975-1416.1029

neutrino experiments to be $sin^2\theta_{14} = 0.019$, $\Delta_{41} = 1.70$ eV^2 [1]. In search of neutrino oscillation, we have obtained the three mass square difference $\triangle m^2_{sun} \ll \triangle m^2_{atm} \ll \triangle m^2_{LSND}$ from atmospheric, solar neutrino and LSND collabration [2–5]. Earlier study of T and CP violation for neutrinos have been given by [6,7]. In this article, we will discuss the possible violation of Time reversal symmetry in four flavour neutrino oscillation. Four Flavour Neutrino Mixing beyond the GUT scale region in Section 2. In Section 3 give the Time reversal symmetry beyond the GUT scale. In Section 4, give the conclusions.

2 Planck Scale Effects in Four Flavour Neutrino Mixing

Neutrino mass squared differences and mixing angles beyond the GUT scale are studieded in earlier paper [8–10]. Mass matrix M of neutrino is given by

$$\mathbf{M} = U^* diag(M_i) U^\dagger, \tag{1}$$

where, M_i, is the neutrino masses and $U_{\alpha i}$ is the usual mixing. Few of the parameters related to neutrino oscillation are known, the major expectation is given by the mixing elements U.

In term of the above mixing angles, the mixing matrix is

$$\begin{aligned} U &= diag(e^{if1}, e^{if2}, e^{if3}, e^{if4}) R(\theta_{34}) R(\theta_{24}) \Delta R(\theta_{14}) \\ &= R(\theta_{13}) R(\theta_{12}) \Delta^* R(\theta_{23}) diag(e^{i\alpha}, e^{i\beta}, e^{i\gamma}, 1). \end{aligned} \tag{2}$$

The Dirac phase associates with the matrix $\Delta = diag(e^{\frac{i\delta}{2}}, 1, 1, e^{\frac{-i\delta}{2}})$. This leads to T/CP violation in neutrino oscillation α, β and γ are the Majorana phases, which effects the neutrinoless double beta decay. f_1, f_2, f_3 and f_4 are the charged mixing angle in the charge lepton field. New mixing matrix beyond the GUT scale is given as [8–10]

$$U' = U(1 + i\delta\theta), \tag{3}$$

where $\delta\theta$ is the form of hermition matrix, the first order neutrino mass square difference $\Delta M'_{ij}$, given by

$$\Delta M'^2_{ij} = \Delta M^2_{ij} + 2.0(-M_j Re(m_{jj}) + M_i Re(m_{ii})), \tag{4}$$

$$m = \mu U^t \lambda U, \tag{5}$$

and

$$\mu = \frac{v^2}{M_{pl}}.$$

The changed mixing matrix is

$$\delta\theta_{ij} = \frac{-Im(m_{ij})(M_i - M_j) + Re(m_{ij})(M_i + M_j)}{\Delta M_{ij}^{\prime 2}}. \tag{6}$$

Using Equation (3), we can compute four flavour neutrino mixing angles [11] as,

$$sin^2\theta'_{14} = |\mathrm{U}'_{\mathrm{s4}}|^2, \tag{7}$$

$$\sin^2\theta'_{24} = \frac{|\mathrm{U}'_{\mathrm{e'4}}|^2}{1 - |\mathrm{U}'_{\mathrm{s4}}|^2}, \tag{8}$$

$$sin^2\theta'_{34} = \frac{|\mathrm{U}'_{\mu 4}|^2}{1 - |\mathrm{U}'_{\mathrm{s4}}|^2 - |\mathrm{U}'_{\mathrm{s4}}|^2}, \tag{9}$$

$$\sin^2\theta'_{13} = \frac{|\mathrm{U}'_{\mathrm{s3}}|^2}{1 - |\mathrm{U}'_{\mathrm{s4}}|^2}, \tag{10}$$

$$sin^2\theta'_{12} = \frac{|\mathrm{U}'_{\mathrm{s2}}|^2}{1 - |\mathrm{U}'_{\mathrm{s4}}|^2 - |\mathrm{U}'_{\mathrm{s3}}|^2}, \tag{11}$$

$$sin^2\theta'_{23} = \frac{|\mathrm{U}'_{\mathrm{e3}}|^2(1 - |\mathrm{U}'_{\mathrm{s4}}|^2) - (|\mathrm{U}'_{\mathrm{s4}}|^2|\mathrm{U}'_{\mathrm{e4}}|^2)}{1 - |\mathrm{U}'_{\mathrm{s4}}|^2 - |\mathrm{U}'_{\mathrm{e4}}|^2} + \frac{|U'_{s1}U'_{e1} + U'_{s2}U'_{e2}|^2(1 - |U'_{s4}|^2)}{(1 - |U'_{s4}|^2 - |U'_{s3}|^2)(1 - |U'_{s4}|^2 - |U'_{e4}|^2)}. \tag{12}$$

where,

$$U'_{\alpha 1} = U_{\alpha 1} + \sum_i U_{\alpha i}\left(\frac{-Re(m_{i1})(M_i + M_1) - iIm(m_{i1})(M_i - M_1)}{M_i^2 - M_1^2 + 2(M_iRe(m_{ii}) - M_1Re(m_{11})}.\right),$$

$$U'_{\alpha 2} = U_{\alpha 2} + \sum_i U_{\alpha i}\left(\frac{-Re(m_{i2})(M_i + M_2) - iIm(m_{i2})(M_i - M_2)}{M_i^2 - M_2^2 + 2(M_iRe(m_{ii}) - M_4Re(m_{22})}.\right),$$

$$U'_{\alpha 3} = U_{\alpha 3} + \sum_i U_{\alpha i}\left(\frac{-Re(m_{i3})(M_i + M_3) - iIm(m_{i3})(M_i - M_3)}{M_i^2 - M_3^2 + 2(M_iRe(m_{ii}) - M_4Re(m_{33})}.\right),$$

$$\mathrm{U}'_{\alpha 4} = \mathrm{U}_{\alpha 4} + \sum_{\mathrm{i}} \mathrm{U}_{\alpha \mathrm{i}} \left(\frac{-\mathrm{Re}(\mathrm{m_{i4}})(\mathrm{M_i} + \mathrm{M_4}) - \mathrm{i}\mathrm{Im}(\mathrm{m_{i4}})(\mathrm{M_i} - \mathrm{M_4})}{\mathrm{M_i^2} - \mathrm{M_4^2} + 2(\mathrm{M_i}\mathrm{Re}(\mathrm{m_{ii}}) - \mathrm{M_4}\mathrm{Re}(\mathrm{m_{44}})}. \right),$$

$$\alpha = \mathrm{s}, \mathrm{e}, \mu, \tau$$

3 Time Reversal Symmetry due to Planck Scale Effects for Four Flavour Mixing

As far on T violation effects in four flavour framework, we find that a comparison of $\nu_\alpha \to \nu_\beta$ and $\nu_\beta \to \nu_\alpha$ oscillation probability. Time reversal symmetry is violated, if

$$\Delta P^T_{\alpha\beta} = P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha) \neq 0, \tag{13}$$

and

$$(\alpha, \beta) = (e, \mu), (\mu, \tau), (\tau, e).$$

P$(\nu_\alpha \to \nu_\beta)$ and $P(\nu_\beta \to \nu_\alpha)$ is oscillation probabilities.

CP violation effects in neutrino oscillations, we find that a comparison of neutrino oscillation $\nu_\alpha \to \nu_\beta$ and $\nu_{\overline{\alpha}} \to \nu_{\overline{\beta}}$ anti-neutrino oscillation probability. CP symmetry is violated, if

$$\Delta P^{CP}_{\alpha\beta} = P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu_\alpha} \to \bar{\nu_\beta}) \neq 0. \tag{14}$$

and

$$(\alpha, \beta) = (e, \mu), (\mu, \tau), (\tau, e).$$

$\Delta P^T_{\alpha\beta}$ and $\Delta P^{CP}_{\alpha\beta}$ defined in Equations (13) and (14) are equal and given by

$$\Delta P^T_{\alpha\beta} = \Delta P^{CP}_{\alpha\beta} = 16J(sin\Delta_{21} sin\Delta_{32} sin\Delta_{31}). \tag{15}$$

here

$$\Delta_{ij} = 1.27 \left(\frac{\Delta_{ij}}{eV^2}\right)\left(\frac{L}{Km}\right)\left(\frac{1GeV}{E}\right), \tag{16}$$

$\Delta_{ij} = (m_i^2 - m_j^2)$ is neutrino mass square difference, L is baseline length, E is energy and J is the Jarlskog determinant [13] is given by

$$\begin{aligned} J &= Im(U_{e1}U^*_{e2}U^*_{\mu 1}U_{\mu 2}) \\ &= \frac{1}{8} sin2\theta_{12} sin2\theta_{23} sin2\theta_{13} cos\theta_{13} sin\delta, \end{aligned} \tag{17}$$

Let us compute $\Delta P^{T}_{\alpha\beta}$ and $\Delta P^{CP}_{\alpha\beta}$ for mixing $U' = U(1 + i\delta\theta)$.

$$\Delta' P^{T}_{\alpha\beta} = \Delta' P^{CP}_{\alpha\beta} = 16J'(sin\Delta'_{21} sin\Delta'_{32} sin\Delta'_{31}), \tag{18}$$

where J' is the Jarlskog determiant for new mixing is given by [13]

$$\begin{aligned} J' &= Im(U'_{e1} U'^{*}_{e2} U'^{*}_{\mu 1} U'_{\mu 2}) \\ &= Im(U_{e1} U^{*}_{e2} U^{*}_{\mu 1} U_{\mu 2}) + Im(i(U_{\mu 1} U_{\mu 2})(|U_{e2}|^2 \delta\theta^{*}_{12} + U_{e2} U_{e3} \delta\theta_{13} \\ &\quad - |U_{e1}|^2 \delta\theta^{*}_{12} - U_{e1} U^{*}_{e3} \delta\theta^{*}_{23}) + Im(i(U^{*}_{e1} U_{e2})(|U_{\mu 1}|^2 \delta\theta_{12} \\ &\quad + U^{*}_{\mu 1} U_{\mu 3} \delta\theta^{*}_{23} - |U_{\mu 2}|^2 \delta\theta_{12} - U_{\mu 2} U^{*}_{\mu 3} \delta\theta_{13}) \\ &= J + \Delta J \end{aligned}$$

The calculation of J Jarlskog determiant [12] for four flavour neutrino oscilation due to Planck scale region. which is given by replacing the neutrino matrix U by new neutrino matrix U',

$$\begin{aligned} J^{13'}_{se} &= Im\Bigg(\Big(U_{s1} + i\sum_i U_{si}\delta\theta_{i1}\Big)\Big(U_{e3} + i\sum_i U_{ei}\delta\theta_{i3}\Big) \\ &\quad \times \Big(U^{*}_{s3} - i\sum_i U^{*}_{ei}\delta\theta^{*}_{i3}\Big)\Big(U^{*}_{e1} - i\sum_i U^{*}_{ei}\delta\theta^{*}_{i1}\Big)\Bigg) \\ J^{24'}_{se} &= Im\Bigg(\Big(U_{s2} + i\sum_i U_{si}\delta\theta_{i2}\Big)\Big(U_{e4} + i\sum_i U_{ei}\delta\theta_{i4}\Big) \\ &\quad \times \Big(U^{*}_{s4} - i\sum_i U^{*}_{ei}\delta\theta^{*}_{i4}\Big)\Big(U^{*}_{e2} - i\sum_i U^{*}_{ei}\delta\theta^{*}_{i2}\Big)\Bigg) \\ J^{34'}_{se} &= Im\Bigg(\Big(U_{s3} + i\sum_i U_{si}\delta\theta_{i3}\Big)\Big(U_{e4} + i\sum_i U_{ei}\delta\theta_{i4}\Big) \\ &\quad \times \Big(U^{*}_{s4} - i\sum_i U^{*}_{ei}\delta\theta^{*}_{i4}\Big)\Big(U^{*}_{e3} - i\sum_i U^{*}_{ei}\delta\theta^{*}_{i3}\Big)\Bigg) \end{aligned}$$

$$
\begin{aligned}
J_{\tau s}^{13'} &= Im\Bigg(\left(U_{\tau 1} + i\sum_i U_{\tau i}\delta\theta_{i1}\right)\left(U_{s3} + i\sum_i U_{ei}\delta\theta_{i3}\right) \\
&\quad \times \left(U_{\tau 3}^* - i\sum_i U_{\tau i}^*\delta\theta_{i1}^*\right)\left(U_{s1}^* - i\sum_i U_{si}^*\delta\theta_{i1}^*\right)\Bigg) \\
J_{\tau s}^{14'} &= Im\Bigg(\left(U_{\tau 1} + i\sum_i U_{\tau i}\delta\theta_{i1}\right)\left(U_{s4} + i\sum_i U_{ei}\delta\theta_{i4}\right) \\
&\quad \left(U_{\tau 4}^* - i\sum_i U_{\tau i}^*\delta\theta_{i4}^*\right)\left(U_{s1}^* - i\sum_i U_{si}^*\delta\theta_{i1}^*\right)\Bigg) \\
J_{\tau s}^{34'} &= Im\Bigg(\left(U_{\tau 3} + i\sum_i U_{\tau i}\delta\theta_{i3}\right)\left(U_{s4} + i\sum_i U_{ei}\delta\theta_{i4}\right) \\
&\quad \left(U_{\tau 4}^* - i\sum_i U_{\tau i}^*\delta\theta_{i4}^*\right)\left(U_{s3}^* - i\sum_i U_{si}^*\delta\theta_{i3}^*\right)\Bigg) \\
J_{e\mu}^{23'} &= Im\Bigg(\left(U_{e2} + i\sum_i U_{ei}\delta\theta_{i2}\right)\left(U_{\mu 3} + i\sum_i U_{\mu i}\delta\theta_{i3}\right) \\
&\quad \times \left(U_{e3}^* - i\sum_i U_{ei}^*\delta\theta_{i3}^*\right)\left(U_{\mu 2}^* - i\sum_i U_{\mu i^*}\delta\theta_{i2}^*\right)\Bigg) \\
J_{e\mu}^{24'} &= Im\Bigg(\left(U_{e2} + i\sum_i U_{ei}\delta\theta_{i2}\right)\left(U_{\mu 4} + i\sum_i U_{\mu i}\delta\theta_{i4}\right) \\
&\quad \times \left(U_{e4}^* - i\sum_i U_{ei}^*\delta\theta_{i4}^*\right)\left(U_{\mu 2}^* - i\sum_i U_{\mu i}^*\delta\theta_{i2}^*\right)\Bigg) \\
J_{e\mu}^{34'} &= Im\Bigg(\left(U_{e3} + i\sum_i U_{ei}\delta\theta_{i3}\right)\left(U_{\mu 4} + i\sum_i U_{\mu i}\delta\theta_{i4}\right) \\
&\quad \times \left(U_{e4}^* - i\sum_i U_{ei}^*\delta\theta_{i4}^*\right)\left(U_{\mu 3}^* - i\sum_i U_{\mu i}^*\delta\theta_{i3}^*\right)\Bigg) \qquad (19)
\end{aligned}
$$

In term of mixing angle and Dirac phases, we can write Jarlskog determinant $J^{ij'}_{\alpha\beta}$ due to Planck scale are,

$$\begin{aligned} J^{13'}_{se} &= \frac{1}{16} sin2\theta'_{12} sin2\theta'_{13} sin2\theta'_{14} sin2\theta'_{24} cos\theta'_{14} cos\theta'_{23} sin\theta'_{13} sin\theta_y \\ &- \frac{1}{16} sin2\theta'_{12} sin2\theta'_{13} sin2\theta'_{14} sin2\theta'_{23} cos\theta'_{14} cos^2\theta'_{24} sin\theta_z \\ &+ \frac{1}{8} sin2\theta'_{13} sin2\theta'_{14} sin2\theta'_{24} cos^2\theta'_{12} cos\theta'_{14} sin\theta'_{23} sin(\theta_y - \theta_z) \end{aligned} \quad (20)$$

$$\begin{aligned} J^{24'}_{se} &= \frac{1}{8} sin2\theta'_{12} sin2\theta'_{14} sin2\theta'_{24} cos\theta'_{13} cos\theta'_{14} cos\theta'_{23} sin\theta_y \\ &+ \frac{1}{4} sin2\theta'_{13} sin2\theta'_{14} sin^2\theta'_{12} cos\theta'_{14} sin\theta'_{24} sin\theta'_{23} sin(\theta_y - \theta_z) \end{aligned} \quad (21)$$

$$J^{34'}_{se} = \frac{1}{8} sin2\theta'_{13} sin2\theta'_{14} sin2\theta'_{24} cos\theta'_{14} sin\theta'_{23} sin(\theta_y - \theta_z) \quad (22)$$

$$\begin{aligned} J^{13'}_{\tau s} &= \frac{1}{8} sin2\theta'_{13} sin2\theta'_{14} sin2\theta'_{34} cos^2\theta'_{12} cos\theta'_{14} cos\theta'_{23} cos\theta'_{24} sin\theta_x \\ &+ \frac{1}{16} sin2\theta'_{12} sin2\theta'_{13} sin2\theta'_{14} sin2\theta'_{24} cos\theta'_{14} cos\theta'_{23} cos^2\theta'_{34} sin\theta_y \\ &+ \frac{1}{8} (cos^2\theta'_{34} sin^2\theta'_{24} - sin^2\theta'_{34}) \\ &\times sin2\theta'_{12} sin2\theta'_{13} sin2\theta'_{23} cos\theta'_{13} cos^2\theta'_{14} sin\theta_z \\ &- \frac{1}{8} sin2\theta'_{12} sin2\theta'_{13} sin2\theta'_{34} cos\theta'_{13} cos^2\theta'_{14} cos^2\theta'_{23} sin\theta'_{24} \\ &\times sin(\theta_x - \theta_y) \\ &- \frac{1}{16} sin2\theta'_{12} sin2\theta'_{13} sin2\theta'_{14} sin2\theta'_{34} cos\theta'_{14} cos\theta'_{24} \\ &\times sin\theta'_{13} sin\theta'_{23} sin(\theta_x + \theta_z) \\ &+ \frac{1}{8} sin2\theta'_{13} sin2\theta'_{14} sin2\theta'_{24} cos^2\theta'_{12} cos\theta'_{14} cos^2\theta'_{34} \end{aligned}$$

$$\times sin\theta'_{23} sin(\theta_y - \theta_z)$$
$$- \frac{1}{8} sin2\theta'_{12} sin2\theta'_{13} sin2\theta'_{34} cos^2\theta'_{14} cos\theta'_{13} sin^2\theta'_{23}$$
$$\times sin\theta'_{24} sin(\theta_x - \theta_y + \theta_z) \tag{23}$$

$$J^{14'}_{\tau s} = -\frac{1}{8} sin2\theta'_{13} sin2\theta'_{14} sin2\theta'_{34} cos^2\theta'_{12} cos\theta'_{14} cos\theta'_{23} cos\theta'_{24} sin\theta_x$$
$$- \frac{1}{8} sin2\theta'_{12} sin2\theta'_{13} sin2\theta'_{24} cos^2\theta'_{14} cos\theta'_{23} cos^2\theta'_{34} sin\theta_y$$
$$- \frac{1}{8} sin2\theta'_{12} sin2\theta'_{14} sin2\theta'_{34} cos\theta'_{13} cos\theta'_{14} cos\theta'_{24} sin\theta'_{23} sin(\theta_x + \theta_z)$$
$$- \frac{1}{8} sin2\theta'_{12} sin2\theta'_{13} sin2\theta'_{24} cos^2\theta'_{14} cos\theta'_{12} cos^2\theta'_{34}$$
$$\times sin\theta'_{23} sin(\theta_y - \theta_z) \tag{24}$$

$$J^{34'}_{\tau s} = \frac{1}{8} sin2\theta'_{13} sin2\theta'_{14} sin2\theta'_{34} cos\theta'_{14} cos\theta'_{23} cos\theta'_{24} sin\theta_x$$
$$+ \frac{1}{8} sin2\theta'_{13} sin2\theta'_{14} sin2\theta'_{24} cos\theta'_{14} cos^2\theta'_{34} sin\theta'_{23} sin(\theta_y - \theta_z) \tag{25}$$

$$J^{23'}_{e\mu} = -\frac{1}{4}(cos^2\theta'_{12} cos^2\theta'_{23} sin^2\theta'_{24} - cos^2\theta'_{12} cos^2\theta'_{24} sin^2\theta'_{23}$$
$$+ sin^2\theta'_{12} sin^2\theta'_{14} sin^2\theta'_{24}$$
$$+ sin^2\theta'_{12} sin^2\theta'_{23}) sin2\theta'_{13} sin2\theta'_{34} sin\theta'_{14} cos\theta'_{23} cos\theta'_{24} sin\theta_x$$
$$+ \frac{1}{4}(cos^2\theta'_{13} cos^2\theta'_{34} sin^2\theta'_{23} - cos^2\theta'_{23} cos^2\theta'_{34} sin^2\theta'_{13}$$
$$+ sin^2\theta'_{13} sin^2\theta'_{14} sin^2\theta'_{34}$$
$$- sin^2\theta'_{34} sin^2\theta'_{23}) sin2\theta'_{12} sin2\theta'_{24} sin\theta'_{14} cos\theta'_{13} cos\theta'_{23} cos\theta'_{24} sin\theta_y$$
$$+ \frac{1}{8}(cos^2\theta'_{24} cos^2\theta'_{34} - cos^2\theta'_{24} sin^2\theta'_{14} sin^2\theta'_{34}$$
$$- cos^2\theta'_{34} sin^2\theta'_{14} sin^2\theta'_{24} + sin^2\theta'_{14} sin^2\theta'_{24} sin^2\theta'_{34})$$
$$\times sin2\theta'_{12} sin2\theta'_{13} sin2\theta'_{23} cos\theta'_{13} cos\theta'_{34} sin\theta_z$$

$$+\frac{1}{4}(sin^2\theta'_{34}cos^2\theta'_{13}-cos^2\theta'_{24}sin^2\theta'_{13})sin2\theta'_{12}sin2\theta'_{34}$$

$$\times cos^2\theta'_{23}sin\theta'_{13}sin^2\theta'_{14}sin\theta'_{24}sin(\theta_x-\theta_y)$$

$$+\frac{1}{4}(cos^2\theta'_{23}-sin^2\theta'_{24}cos^2\theta'_{13}-cos^2\theta'_{24}sin^2\theta'_{13}$$

$$+sin^2\theta'_{13}sin^2\theta'_{14}sin^2\theta'_{24})$$

$$\times sin2\theta'_{12}sin2\theta'_{34}cos\theta'_{13}cos\theta'_{24}sin\theta'_{23}sin(\theta_x+\theta_z)$$

$$-\frac{1}{4}(cos^2\theta'_{12}cos^2\theta'_{23}cos^2\theta'_{34}-cos^2\theta'_{12}cos^2\theta'_{23}sin^2\theta'_{34}$$

$$-cos^2\theta'_{23}cos^2\theta'_{34}sin^2\theta'_{12}+sin^2\theta'_{12}sin^2\theta'_{14}sin^2\theta'_{34}$$

$$-sin^2\theta'_{12}sin^2\theta'_{23}sin^2\theta'_{34})sin2\theta'_{13}sin2\theta'_{24}sin\theta'_{14}sin\theta'_{23}sin(\theta_y-\theta_z)$$

$$+\frac{1}{4}(cos^2\theta'_{12}cos^2\theta'_{13}cos^2\theta'_{24}-cos^2\theta'_{12}cos^2\theta'_{24}sin^2\theta'_{13}sin^2\theta'_{14}$$

$$-cos^2\theta'_{13}cos^2\theta'_{24}sin^2\theta'_{12}sin^2\theta'_{14}+cos^2\theta'_{24}sin^2\theta'_{12}sin^2\theta'_{13}sin^2\theta'_{14})$$

$$\times sin2\theta'_{23}sin2\theta'_{34}sin\theta'_{23}sin(\theta_x-\theta_y+\theta_z)$$

$$-\frac{1}{4}(cos^2\theta'_{13}cos^2\theta'_{24}sin^2\theta'_{23}-cos^2\theta'_{13}sin^2\theta'_{23}sin^2\theta'_{24}sin^2\theta'_{14}$$

$$-cos^2\theta'_{24}sin^2\theta'_{23}sin^2\theta'_{13}sin^2\theta'_{14})$$

$$\times sin2\theta'_{12}sin2\theta'_{34}sin\theta'_{13}sin\theta'_{24}sin(\theta_x-\theta_y+2\theta_z)$$

$$-\frac{1}{16}sin2\theta'_{12}sin2\theta'_{13}sin2\theta'_{24}sin2\theta'_{34}cos\theta'_{13}cos\theta'_{24}sin^2\theta'_{14}$$

$$\times sin(\theta_x+\theta_y)$$

$$+\frac{1}{16}sin2\theta'_{12}sin2\theta'_{13}sin2\theta'_{23}sin2\theta'_{34}sin\theta'_{13}cos\theta'_{24}$$

$$\times sin\theta'_{14}sin(\theta_x-\theta_z)$$

$$+\frac{1}{8}(cos^2\theta'_{34}-sin^2\theta'_{34})sin2\theta'_{24}sin2\theta'_{23}sin2\theta'_{34}sin\theta'_{13}cos\theta'_{12}$$

$$\times sin\theta'_{24}sin\theta'_{14}sin\theta'_{23}sin(\theta_y-2\theta_z)$$

$$
\begin{aligned}
&-\frac{1}{8}(cos^2\theta'_{12}-sin^2\theta'_{12})sin2\theta'_{13}sin2\theta'_{23}sin2\theta'_{34}sin^2\theta'_{24}\\
&\times cos\theta'_{24}sin\theta'_{14}sin\theta'_{23}sin(\theta_x-2\theta_y+2\theta_z)\\
&-\frac{1}{8}sin2\theta'_{12}sin2\theta'_{23}sin2\theta'_{34}cos\theta'_{12}cos\theta'_{23}sin\theta'_{24}cos\theta'_{34}sin\theta'_{14}\\
&\times sin\theta'_{34}sin(\theta_x-2\theta_y+\theta_z)\\
&+\frac{1}{16}sin2\theta'_{12}sin2\theta'_{13}sin2\theta'_{24}sin2\theta'_{34}sin\theta'_{13}sin\theta'_{14}sin^2\theta'_{23}\\
&\times sin\theta'_{24}sin(\theta_x-2\theta_y+3\theta_z)
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
J^{24'}_{e\mu}=&\frac{1}{16}sin2\theta'_{13}sin2\theta'_{14}sin2\theta'_{24}sin2\theta'_{34}cos\theta'_{14}cos\theta'_{23}sin\theta'_{14}sin\theta'_{24}sin\theta_x\\
&+\frac{1}{8}sin2\theta'_{12}sin2\theta'_{14}sin2\theta'_{24}cos\theta'_{13}cos\theta'_{14}cos\theta'_{23}sin^2\theta'_{34}sin\theta_x\\
&-\frac{1}{4}sin2\theta'_{12}sin2\theta'_{24}cos^2\theta'_{14}cos^2\theta'_{23}cos\theta'_{24}cos\theta'_{34}sin\theta'_{13}\\
&\times sin\theta'_{34}sin(\theta_x-\theta_y)\\
&+\frac{1}{16}sin2\theta'_{12}sin2\theta'_{14}sin2\theta'_{24}sin2\theta'_{34}cos\theta'_{13}sin\theta'_{23}\\
&\times sin\theta'_{24}sin(\theta_x+\theta_y)\\
&-\frac{1}{8}sin2\theta'_{13}sin2\theta'_{14}sin2\theta'_{24}cos\theta'_{14}sin^2\theta'_{12}sin\theta'_{23}\\
&\times sin^2\theta'_{34}sin(\theta_y-\theta_z)\\
&+\frac{1}{8}sin2\theta'_{12}sin2\theta'_{13}sin2\theta'_{34}cos\theta'_{13}cos^2\theta'_{14}sin^2\theta'_{23}\\
&\times sin\theta'_{24}sin(\theta_x-\theta_y+2\theta_z)\\
&-\frac{1}{4}(cos^2\theta'_{12}-sin^2\theta'_{12}sin^2\theta'_{13})sin2\theta'_{23}sin2\theta'_{24}cos^2\theta'_{14}cos\theta'_{34}\\
&\times cos\theta'_{24}sin\theta'_{34}sin(\theta_x-\theta_y+\theta_z)
\end{aligned}
\tag{27}
$$

$$\begin{aligned}
J_{e\mu}^{34'} &= -\frac{1}{16} sin2\theta'_{13} sin2\theta'_{14} sin2\theta'_{24} sin2\theta'_{34} cos\theta'_{14} cos\theta'_{23} sin\theta'_{24} sin\theta_x \\
&+ \frac{1}{8} sin2\theta'_{13} sin2\theta'_{14} sin2\theta'_{24} cos\theta'_{14} sin\theta'_{23} sin^2\theta'_{34} sin(\theta_y - \theta_z) \\
&+ \frac{1}{4} sin2\theta'_{23} sin2\theta'_{24} cos^2\theta'_{13} cos^2\theta'_{14} cos\theta'_{24} cos\theta'_{34} \\
&\times sin\theta'_{34} sin(\theta_z + \theta_x - \theta_y) \qquad (28)
\end{aligned}$$

$$\begin{aligned}
\theta_x &= \delta_{14} - (\delta_{13} + \delta_{34}) \\
\theta_y &= \delta_{14} - (\delta_{12} + \delta_{24}) \\
\theta_z &= \delta_{13} - (\delta_{12} + \delta_{23}) \qquad (29)
\end{aligned}$$

4 Conclusions

We discussed some importance of T violation in four flavour neutrino oscillation beyond the GUT scale. We have presented four flavour neutrino mixing and possible T violation term $\Delta' P^T_{\alpha\beta}$ above the GUT scale. In four flavour neutrino oscillation above the GUT scale region [10]. The mixing angle changes in θ_{14}, θ_{24} and θ_{34} above the GUT scale, are very small. But the change in θ_{23} is very large for large range of values of the majaorona phases α, β and γ. In four flavour mixing gives the range of mixing angle $\theta'_{12} = \theta_{12} \pm 3.0°$, $\theta'_{12} = \theta_{12} \pm 45°$ [12] modified mass square difference $\Delta'_{21} = \Delta_{21} \pm (1.0 + 0.5) \times 10^{-5}$ eV^2 [10], for Planck scale $M_{pl} \approx 2.0 \times 10^{19}$ GeV. In this study, beyond the GUT scale region, we have obtained, solar mixing angle θ_{12}, atmoshpheric angle θ_{23} and solar neutrino mass square difference Δ'_{21} are more effective for Time Reversal symmetry violation. We would like to conclude that in planck scale region, two mixing angle θ_{12}, θ_{23} and solar mass square difference Δ'_{21} will more effective for Time Reversal symmetry violation.

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