Journal of Graphic Era University
Vol. 7, Issue 2, 132-152, 2019
ISSN: 0975-1416 (Print), 2456-4281 (Online)

Design and Analysis of Robust Multivariable PI Controller for Paper Machine Headbox Using Particle Swarm Optimisation

Parvesh Saini[1*], Rajesh Kumar[2]

[1]Department of Electrical Engineering
[2]Department of Electronics Engineering
Graphic Era (Deemed to be University), Dehradun, Uttarakhand, India
[*]Corresponding author: parvesh.saini.eee@geu.ac.in, parv1606@gmail.com

(Received March 31, 2019; Accepted July 27, 2019)

Abstract
Most of the control loops in paper machine are multivariable loops. Due to multivariable nature of the loops, there is probability that loop have significant interaction. So, the prime objective of designing controllers for multivariable systems is that the process variations are minimized to get desired response. This paper presents a multi-variable control system approach to design PI controller using Particle Swarm Optimisation (PSO) for paper machine headbox. Paper machine headbox is a 2-input and 2-output sub-process of paper machine. The major parameters to be controlled in headbox are Total Head (Pressure) and Stock Level. The performance of PSO based multivariable PI controller has been compared with three conventional controller tuning techniques. The performance assessment of multivariable controllers has been done on the basis of time response, frequency response, performance indices and robustness.

Keywords- Paper Machine Headbox, Multivariable Controller, Particle Swarm Optimisation (PSO), Internal Model Control (IMC); Ziegler–Nichols (ZN), Tyreus-Luyben (TL).

1. Particle Swarm Optimisation (PSO)

Like other Evolutionary Algorithms (EA), Particle Swarm Optimization (PSO) is also an optimisation technique proposed by Kennedy and Eberhart (Kennedy and Eberhart, 1995). This technique was developed through simulation of artificial livelihoods. “Particle” term used in PSO indicates birds, fish, bees or other likely agents, which exhibit swarm behavior. PSO is an optimization technique, which is based on population. In PSO, set of feasible solutions is initiated randomly followed by searching of the optimal point. All the particles follow the best particle to determine optimum location. As compared to other EAs, PSO functions in a more intelligent way and conveniently. PSOs have many advantages and due to this this EA technique is used suitably in research areas (El-Shorbagy and Hassanien, 2018). PSO fits to Swarm Intelligence group, to cater to solution of global optimisation problems (Eberhart et al., 2001). Some of the major advantages of PSO is that it is easy to implement, its computation cost is low, it needs low memory and processing speed. Additionally, PSO technique only needs the value of objective function (Eberhart et al., 1996).

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2. Headbox

The important subsystem of paper machine is headbox (also known as flow box) which is used to spread pulp uniformly over the wire (Xiao and Wang, 2009). It is a highly nonlinear and complex two input - two output system with significant loop interaction. The headbox is subjected to disturbances from pumps, poor tuned controllers and variation in the concentration of the stock and there is strong loop interaction exists between the two loops (Nissinen et al., 1996). Hence, its precise control is highly required to cater the need of better quality paper. For researchers, headbox has been an interesting process to design controller. In past few decades, many control strategies have been developed for paper machine headbox. A brief review of such techniques have been discussed in Saini and Kumar, (2018). Stock level and pressure inside the headbox are two major parameters which have to be controlled. Proper control of a system can be ensured only through its perfect mathematical modeling. For this paper, air cushioned pressurized headbox (Paattilammi and Makila, 2000) has been considered. A schematic of headbox has been shown in Figure 1.

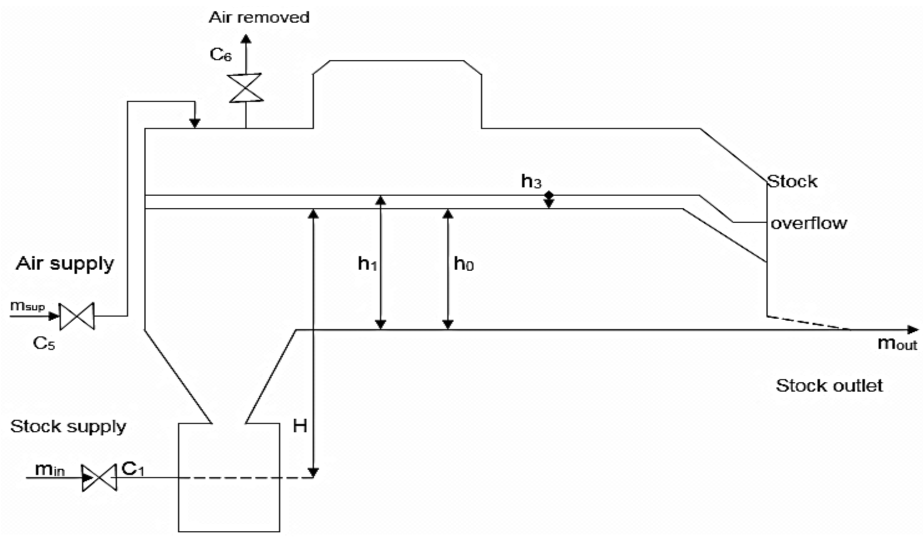

Figure 1. Air cushioned pressurized headbox (Saini and Kumar, 2018)

The objective of this paper is to design a multivariable (MIMO) PI controller using PSO technique for paper machine headbox and compare the performance with the conventional controller tuning techniques. MIMO control methodology is different from decoupled technique in a way that instead of using decouplers, each process element uses a controller which are connected in MIMO loops. In this work, instead of decoupling control, MIMO control has been used. The following section gives a brief description of mathematical model of headbox.

3. Particle Swarm Optimisation Algorithm

This section presents the PSO algorithm. Figure 2 represents the block diagram of PI controller tuning using PSO.

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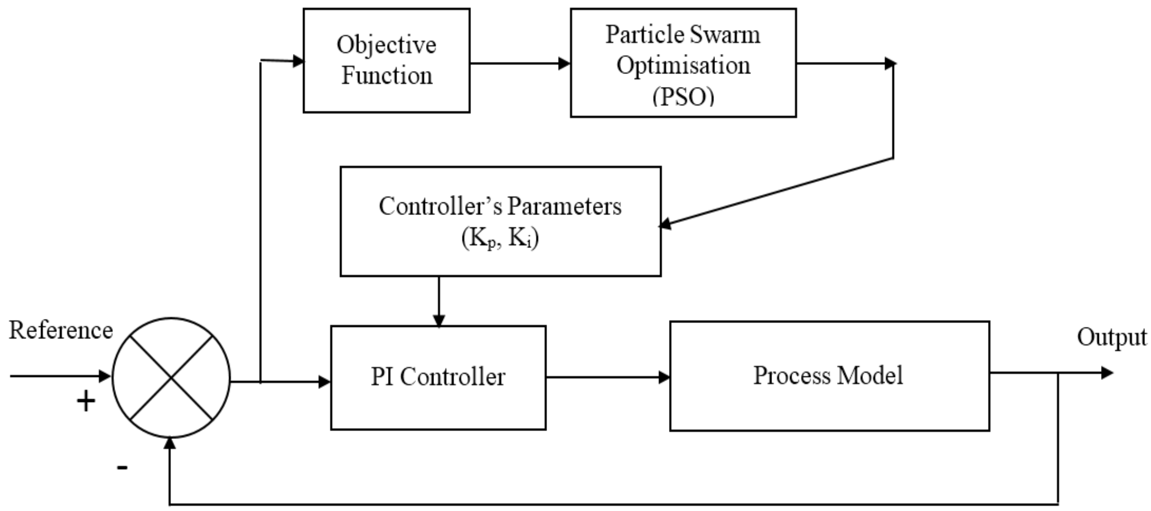

Figure 2. Block diagram of PI tuning using PSO

The particle velocity equation is given as:

$$\upsilon_{n+1}^{j} = \omega\upsilon_{n}^{j} + d_1 p_1\left(r_n^j - y_n^j\right) + d_2 p_2\left(r_n^k - y_n^j\right) \tag{1}$$

Where, "ω" represents inertia weight that performs scaling function of the particle velocity (υ_n). Greater value of "ω" lead to quality search with better characteristics. Stepwise PSO algorithm is discussed below (Kennedy and Eberhart, 1995; El-Shorbagy and Hassanien, 2018):

Step 1. Initialization

(a) Set constants
(b) Initiate random positioning of particles
(c) Initiate random velocities of particles

Step 2. Optimisation

(a) Evaluation of value of function f_n^j.

(b) Check whether f_n^j is less than or equal to f_{best}^j.

If yes, then $f_{best}^j = f_n^j$, $r_n^j = y_n^j$.

(c) Check whether f_n^j is less than or equal to f_{best}^k.

If yes, then $f_{best}^k = f_n^j$, $r_n^k = y_n^j$.

(d) Go to step 3, in case discontinuing condition is fulfilled.
(e) Updation of all velocities of the particles.
(f) Updation of all locations of particles.
(g) Evaluate: *n = n+1*;

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(h) Move to part a of step 2.

Step 3. Cessation

The constriction factor (Clerc, 1999) is given by:

$$\gamma = \frac{2}{\left|2 - 2\varepsilon - \left(\sqrt{(2\varepsilon)^2 - 8\varepsilon}\right)\right|} \tag{2}$$

Where, "ε = sum of control parameters (c1 and c2) and $\varepsilon > 4$.

4. Headbox Model and Controller Design

The headbox model considered for this work is given in (Nissinen et al., 1996; Paattilammi and Makila, 2000). A 2x2 headbox model is given as:

$$\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \begin{bmatrix} \dfrac{0.528e^{(-0.6s)}}{(2.2s+1)} & \dfrac{(1.2539s+.063)}{(30.051s^2+17.79s+1)} \\ \dfrac{(0.0205s+.000149)e^{(-1.5s)}}{(43.6s^2+s)} & -\dfrac{(0.0007)e^{(-2s)}}{s} \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_1(s) \end{bmatrix}.$$

However, the approximated model of headbox taking important dynamics into consideration is given as:

$$\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \begin{bmatrix} \dfrac{0.528e^{-0.6s}}{2.2s+1} & \dfrac{0.081}{1.89s+1} \\ \dfrac{1.49\times10^{-4}e^{-1.5s}}{s} & \dfrac{-7.0\times10^{-4}e^{-2s}}{s} \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix}$$

$$G_{Headbox}(s) = \begin{bmatrix} \dfrac{0.528e^{-0.6s}}{2.2s+1} & \dfrac{0.081}{1.89s+1} \\ \dfrac{1.49\times10^{-4}e^{-1.5s}}{s} & \dfrac{-7.0\times10^{-4}e^{-2s}}{s} \end{bmatrix} \tag{3}$$

Where, y_1 and y_2 are pressure and stock level in the headbox respectively. And u_1 and $u_2$are the feed pump speed and air valve position respectively. Here, pressure and stock level are the controlled variables. However, feed pump speed and air valve position are the manipulated variables. The process model as given by equation 1 and/or 3 include four process elements which are given by:

$$g_{11} = \frac{0.528e^{-0.6s}}{2.2s+1};\ g_{12} = \frac{0.081}{1.89s+1};\ g_{21} = \frac{1.49\times10^{-4}e^{-1.5s}}{s};\ g_{22} = \frac{-7.0\times10^{-4}e^{2s}}{s}$$

g_{11} is transfer function between *headbox pressure* and *feed pump speed*,

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g_{12} is transfer function between *headbox pressure* and *air valve position,*
g_{21} is transfer function between *headbox stock level* and *feed pump speed,*
g_{22} is transfer function between *headbox stock level* and *air valve position.*

From above all transfer functions, "g_{11}" represents the headbox pressure loop and "g_{22}" represents the stock level loop of the headbox. However, "g_{12}" and "g_{21}" act as disturbances to headbox pressure and stock level respectively.

For a headbox, it is important to maintain the level of the stock and pressure inside it. These two parameters are greatly affected by the variations in the manipulated variables i.e. feed pump speed and air valve position. Air valve position act as disturbance parameter for the headbox pressure and feed pump speed act as disturbance for stock level. So, the variation in above variables further affects the pressure on slice lip and jet velocity which in turn affect the rush - drag ratio. Ideally rush-drag ratio should be 1, however, practically its value must be as near to 1 as possible. So, to maintain the rush-drag ratio, feed pump speed and air valve position need to be manipulated precisely.

The relevant PSO parameters used/obtained for this work to determine controller tuning parameters are depicted in Table 1.

Table 1. PSO parameters

Parameters	**Values**
Swarm Size	50
Step Size	50
Dimension of problem	2 (kp, and ki)
C1	1.2
C2	0.5
Inertia weight (ω)	0.8

PSO algorithm is used to determine the controller's gains (kp and ki) for multivariable PI controller (refer table 2). Controller design using PSO and other conventional techniques has been discussed in next section.

$$K_P = \begin{bmatrix} k_{p11}^{-1} & k_{p12}^{-1} \\ k_{p21}^{-1} & k_{p22}^{-1} \end{bmatrix}^{-1} \tag{4}$$

$$K_I = \begin{bmatrix} k_{i11}^{-1} & k_{i12}^{-1} \\ k_{i21}^{-1} & k_{i22}^{-1} \end{bmatrix}^{-1} \tag{5}$$

The transfer function of MIMO – PI controller is given as:

$$C(s) = K_P + K_I \frac{1}{s} \tag{6}$$

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The matrix of multivariable controller given by equation (12) is

$$C(s)=\begin{bmatrix} C_{11}(s) & C_{12}(s) \\ C_{21}(s) & C_{22}(s) \end{bmatrix} \tag{7}$$

Where, $C_{11}(s)$ and $C_{22}(s)$ are the main controllers for Pressure and Stock Level respectively. While, $C_{12}(s)$ and $C_{21}(s)$ are the cross - controllers for Stock Level and Pressure respectively.

Table 2. Controller's tuning parameters using PSO

Process	Controller's Parameters	
	Kp	Ti
g11	3.754	2.26
g12	30.23	1.86
g21	963.59	180.46
g22	-459.32	403.62

Multivariable PSO – PI controller gain matrices are given below:

$$Kp_{\_PSO} = \begin{bmatrix} 3.5442 & 53.8517 \\ 1.6894 & -433.6542 \end{bmatrix};\; Ki_{\_PSO} = \begin{bmatrix} 1.6265 & 0.1140 \\ 0.3466 & -1.1137 \end{bmatrix} \tag{8}$$

Similarly, using the tuning rules of IMC (Chien, 1990; Garcia and Morari, 1982; Rivera et. al., 1986; Skogestad, 2003), ZN (Ziegler and Nichols, 1942) and TL (Tyreus and Luyben, 1992) for PI controller, the MIMO controllers obtained are given as:

$$Kp\_imc = \begin{bmatrix} 3.64 & 33.15 \\ 1.82 & -408.45 \end{bmatrix};\; Ki\_imc = \begin{bmatrix} 2.26 & 4.33 \\ 1.07 & -45.18 \end{bmatrix} \tag{9}$$

A detailed design of IMC controller for paper machine headbox depicted by equation 3 has been presented by Saini and Kumar (2019).

$$Kp_{\_ZN} = \begin{bmatrix} 5.45 & 11.23 \\ 0.87 & -502.85 \end{bmatrix};\; Ki_{\_ZN} = \begin{bmatrix} 3.006 & 0.21 \\ 0.36 & -75.63 \end{bmatrix} \tag{10}$$

$$Kp_{\_TL} = \begin{bmatrix} 3.783 & 7.80 \\ 0.60 & -349.21 \end{bmatrix};\; Ki_{\_TL} = \begin{bmatrix} 0.79 & 0.055 \\ 0.094 & -19.89 \end{bmatrix} \tag{11}$$

5. Result Analysis

5.1 Result Analysis for Nominal Headbox Model

The designed MIMO PSO - PI controller has been implemented using MATLAB/SIMULINK. Figure 3 depicts the Simulink model of MIMO Controller for headbox model. From this Figure it can be understood that how a MIMO control can be implemented. The MIMO approach is different from decoupling approach of controller design for multivariable system in a way that instead of using decoupler, cross controllers are used. As shown in the Figure, C11 and C22 are main controllers. Whereas; C12 and C21 are cross controllers. This section will present performance assessment of controllers on all four major criterions as discussed in section 4.

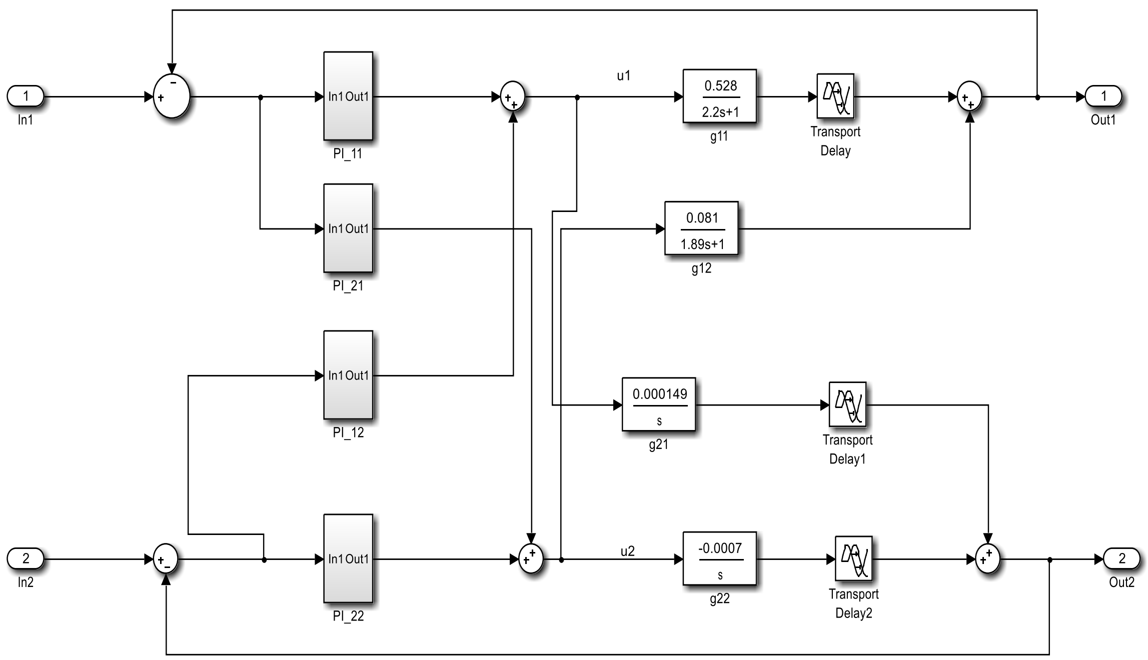

Figure 3. Simulink model of the multivariable PI controller for headbox

5.1.1 Assessment of Performance

This section presents assessment of performance of controllers. The performance of controllers is analysed on the basis of *time response characteristics* of the system. To begin with, closed loop step responses have been assessed. Figure 4 shows the comparison of closed loop step responses of MIMO control system for nominal headbox model.

In this Figure 5, there are four plots which indicate responses for all four process elements (such as g11, g12, g21 and g22). For example, the responses shown in plot *"from r(1)"* to *"pressure"* indicates the closed loop control system for pressure loop. Similarly, the plot *"from r(2)"* to *"stock level"* indicates closed loop control system for stock level loop. Here, "r()" indicates reference signal. Since, the plots are too small to visualize, so the expanded plots of step responses are shown in Figure 5 and 6. Figure 5 represents a comparison of step response of pressure loop and Figure 6 represents a comparison of step response of stock level using all four MIMO PI controllers.

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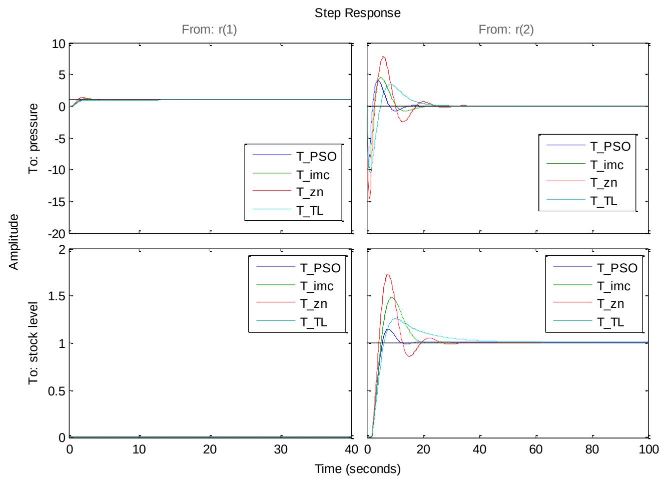

Figure 4. Comparison of step response of nominal headbox model

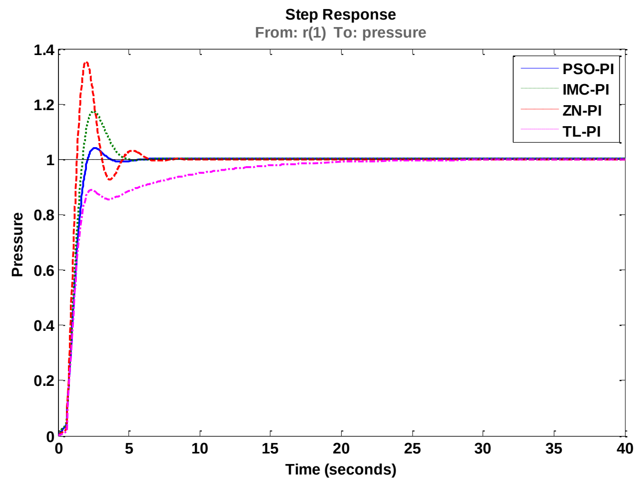

Figure 5. Comparison of step response of nominal headbox pressure

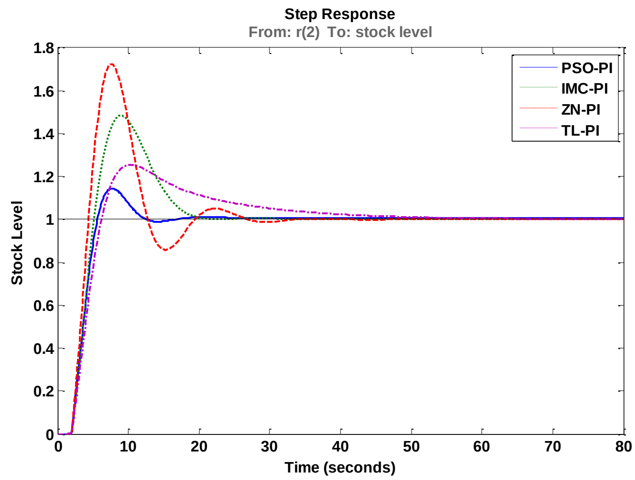

Figure 6. Comparison of step response of nominal headbox stock level

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Vol. 7, Issue 2, 132-152, 2019
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The respective time response values of both plots (Figure 5 and Figure 6) have been presented in Table 3.

Table 3. Comparison of step response values using MIMO PI controller

Process Loop	Tuning Technique	Time Response		
		Rise Time (sec)	Overshoot (%age)	Settling Time (sec)
"Pressure" [From r(1) to Pressure]	PSO	1.08	4.06	3.21
	IMC	0.90	17.38	4.27
	ZN	0.60	35.41	5.78
	TL	5.06	0	15.62
"Stock Level" [From r(2) to Pressure]	PSO	2.77	14.49	11.56
	IMC	2.49	48.42	18.77
	ZN	1.89	72.47	25.29
	TL	3.21	25.41	42.09

From Table 3, it is observed that the rise time from PSO is slightly greater than that obtained using IMC and ZN, but lower than TL for headbox pressure. However, headbox pressure settles down faster for PSO. Also, the peak overshoot is only 4.06% which is acceptable.
Similarly, the PSO MIMO-PI controller yields optimal results for headbox stock level. Rise time due to PSO is less than TL, nearly equal to IMC and greater than ZN. However, peak overshoot and settling time due to PSO are smallest than all three.

Another time response characteristic of headbox model is *load disturbance rejection* (i.e. disturbance appearing at output of system) capability of controllers. It is one of the objectives of controller. Every controller is designed with an aim that if any disturbance appears at the output, it must be suppressed in no time. i.e. time taken to reject disturbance by controller must be as minimum as possible. This is measured in terms of settling time of load disturbance. Settling time is obtained by computing step response of sensitivity function.

Sensitivity function is transfer function between output (y) and load disturbance (d). Mathematically, it is given by:

$$S(s) = \frac{1}{1 + C(s)G(s)} \tag{12}$$

where, *"C(s) – Controller", and "G(s) is the plant"; "C(s)G(s) is loop transfer function".*

Figure 7 depicts load disturbance rejection settling time comparison of pressure loop and Figure 8 depicts the same for stock level. In both Figures, color dots on plots represent settling time values.

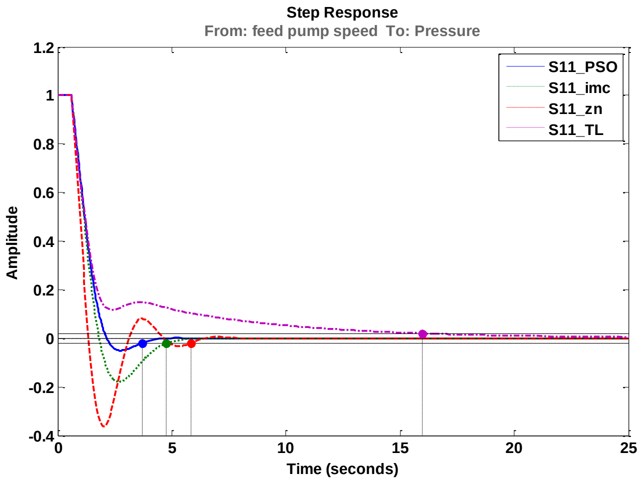

Figure 7. Sensitivity function step response comparison of headbox pressure

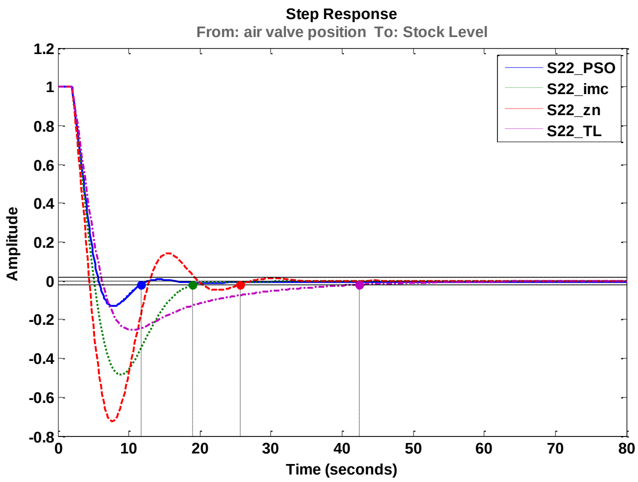

Figure 8. Sensitivity function step response comparison of headbox stock level

The respective values of both plots (Figure 7 and 8) have been given in Table 4. From the Table, it is observed that PSO based MIMO-PI controller suppress the effect of disturbance in system's output faster than other three MIMO-PI controllers for both loops.

Table 4. Load disturbance rejection analysis (Sensitivity Function) using MIMO PI controller

Controller Type	Settling Time (in seconds)	
	Pressure Loop	Stock Level Loop
PSO – PI	3.70	11.67
IMC – PI	4.77	18.99
ZN – PI	5.84	25.62
TL – PI	15.99	42.35

The third criterion for performance assessment of control system is to evaluate performance indices. Table 5 illustrates performance index values for all four MIMO – PI Controllers.

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Vol. 7, Issue 2, 132-152, 2019
ISSN: 0975-1416 (Print), 2456-4281 (Online)

Table 5. Performance indices using MIMO PI controller

Performance Indices	Pressure Loop				Stock Level Loop			
	PSO	IMC	ZN	TL	PSO	IMC	ZN	TL
ISE	0.92	0.91	0.92	1.07	3.14	4.31	4.94	3.86
IAE	1.22	1.37	1.44	2.32	6.8	7.16	7.76	7.90
ITAE	1.1	1.66	1.85	10.05	39.26	45.04	51.79	87.72
ITSE	0.48	0.53	0.58	1.03	7.81	17.19	21.42	14.07

5.1.2 Assessment of Robustness

Robustness of designed control system has been assessed on the basis of *Relative Stability, Sensitivity Function and Complementary Sensitivity Function.* Relative stability of control system is determined from bode plots (or Nyquist plots) by computing stability margins such as Gain Margin and Phase Margin. Stability margins have been obtained from loop transfer function of the given system and are depicted in Table 6.

Table 6. Frequency responses comparison using MIMO PI controller

Process	Tuning Technique	Stability Margins	
		Gain Margin (in dB)	Phase Margin (in degree)
"Pressure" [From r(1) to Pressure]	PSO	9.75	60.5
	IMC	9.05	49.9
	ZN	5.76	40.3
	TL	9.76	76.5
"Stock Level" [From r(2) to Pressure]	PSO	8.26	54.7
	IMC	7.77	35.2
	ZN	5.46	24.9
	TL	9.68	48.5

From the values obtained, (as depicted in Table 6), it is observed that PSO based MIMO-PI controller yields optimal stability margin. These values have been shown graphically in Figure 9 and Figure 10 for headbox pressure and stock level respectively. The notations used (as shown in graphs) are "*L11pso"* for loop 1 (pressure) with PSO PI controller and *"L22pso"* indicates loop 2 (stock level) with PSO PI controller. Similarly, notations for other PI controllers can be understood.

Sensitivity function (S(s)) and complementary sensitivity function (T(s)) analysis have been done by computing peak values of their magnitudes. Magnitudes of S(s) and T(s) are known as amplitude ratio. Figure 11 and Figure 12 depict comparison of sensitivity function analysis of headbox pressure and stock level respectively.

The color dots on both plots indicate the peak values of magnitude plots. The notations used (as shown in graphs) are:

"S*11_PSO"*: S for sensitivity function; 11 for pressure loop and PSO indicates PSO based MIMO PI Controller. Similarly, notations for other PI controllers can be understood.

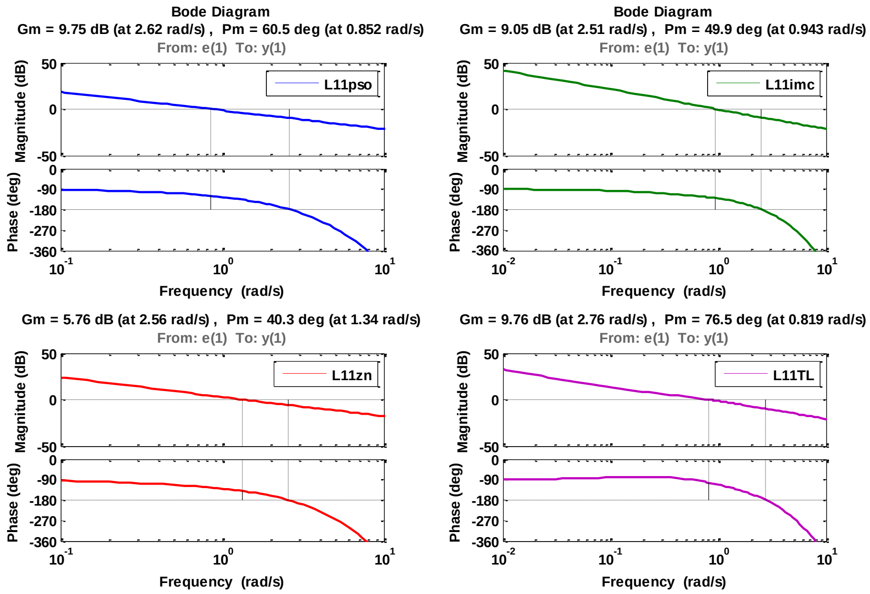

Figure 9. Stability margin comparison of headbox pressure

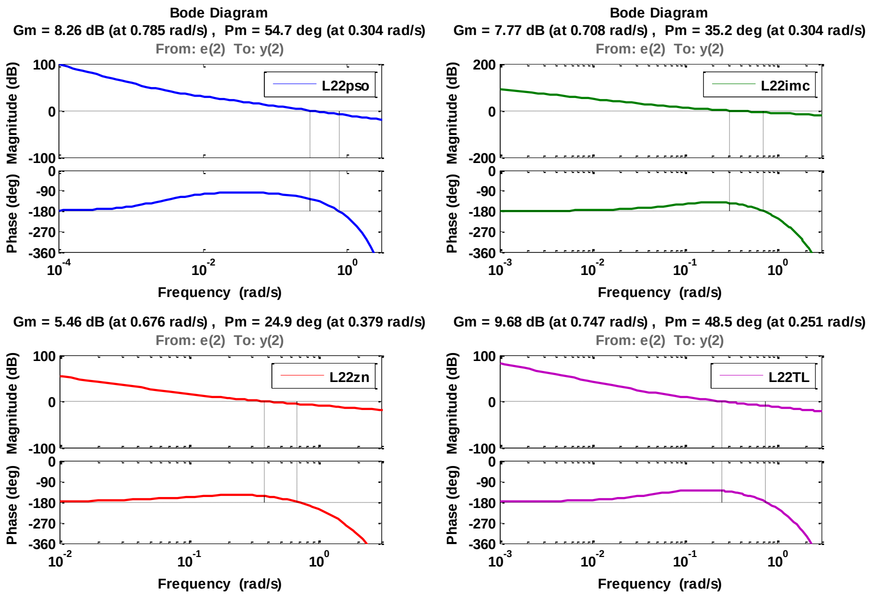

Figure 10. Stability margin comparison of headbox stock level

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ISSN: 0975-1416 (Print), 2456-4281 (Online)

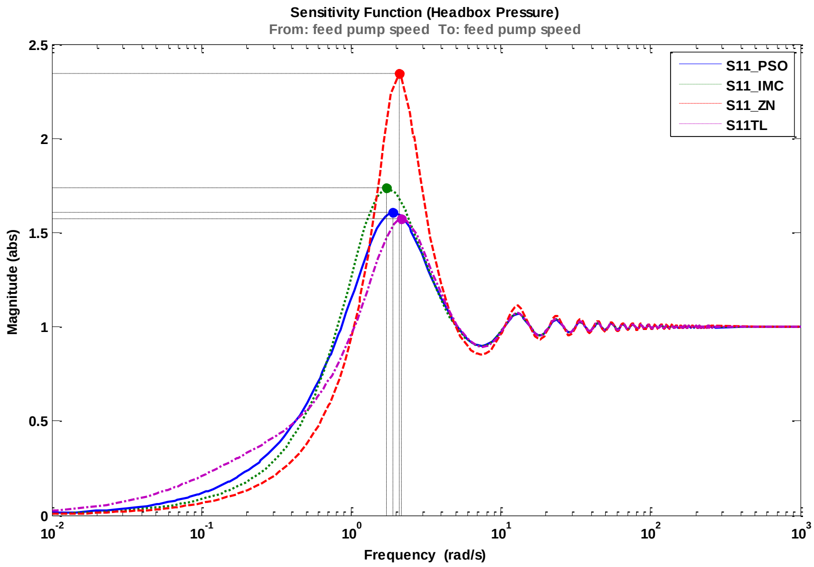

Figure 11. Sensitivity function comparison of headbox pressure

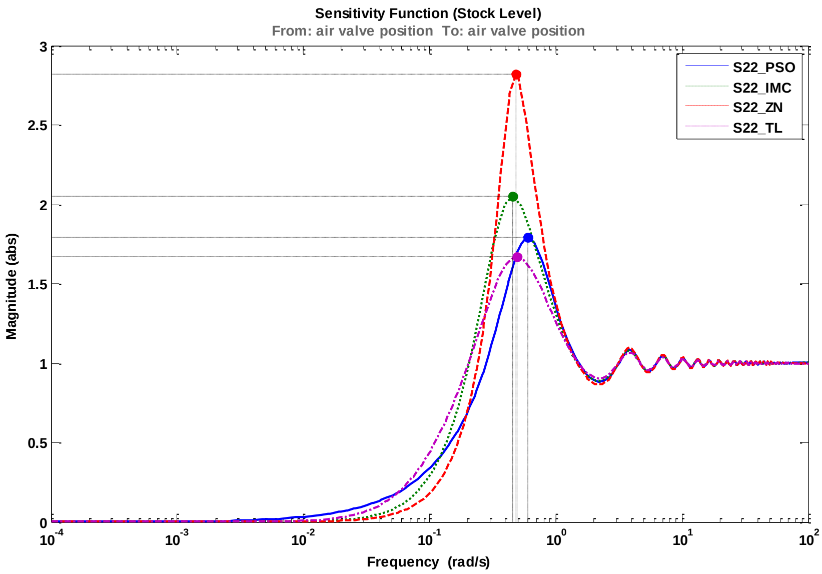

Figure 12. Sensitivity function comparison of headbox stock level

Sensitivity function is an important robustness measure of a system. It indicates the controller's capability to handle disturbance at output. The peak values of magnitude plots of sensitivity function indicate effect of load disturbance on system's response. The normal values of magnitude of S(s) shall exist between 1.2 to 2.0. From the Figures it is observed that PSO based MIMO-PI controller yields optimal results for both loops.

Similarly, the gain plots of complementary sensitivity function for both loops have been depicted in Figure 13 and 14. The notations used (as shown in graphs) are:

"T*11_PSO*": T for complementary sensitivity function; 11 for pressure loop and PSO indicates PSO based MIMO PI Controller. Similarly, notations for other PI controllers can be understood.

The peak values of magnitude plots of T(s) for both loops are indicated by color dots as shown in Figure 13 and 14. The peak value of magnitude of T(s) exist between 1 to 1.5. From both Figures of magnitude plots of T(s), it is observed that PSO based MIMO-PI controller yields optimal results for both loops.

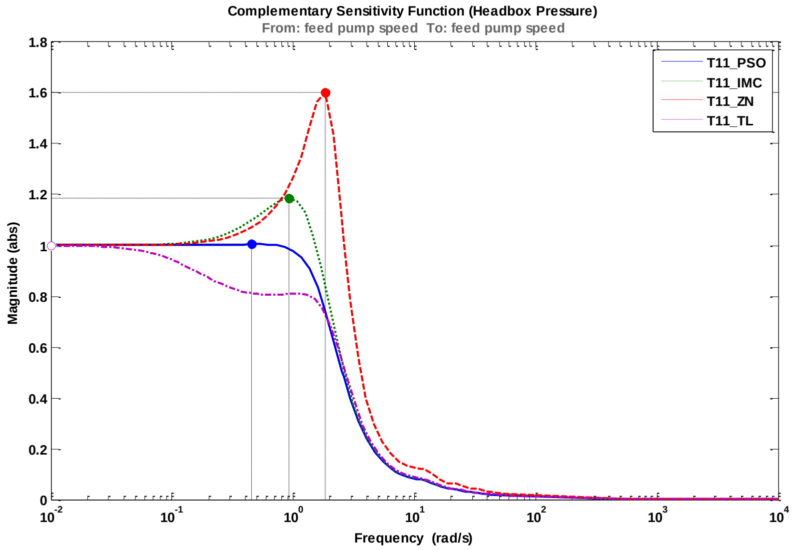

Figure 13. Complementary sensitivity function comparison of headbox pressure

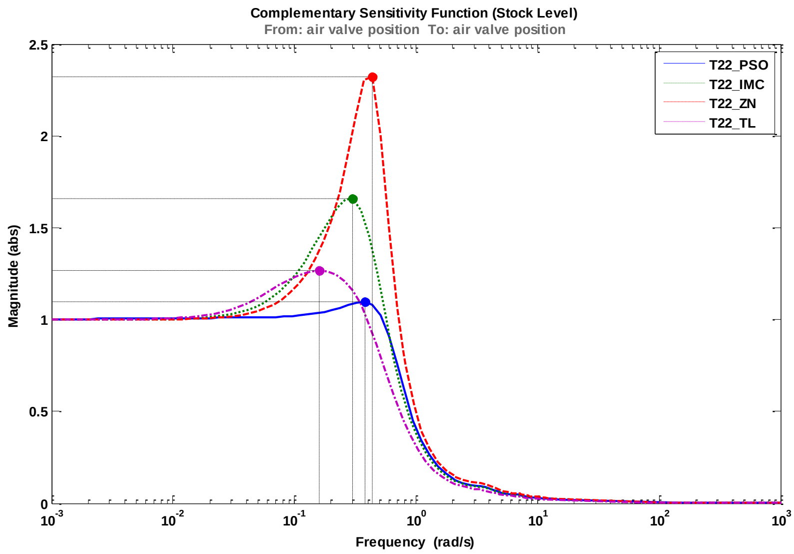

Figure 14. Complementary sensitivity function comparison of headbox stock level

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The respective nominal values of S(s) and T(s) have been shown in Table 7.

Table 7. Nominal peak values of S(s) and T(s)

Controller	Nominal Peak Values			
	S(s)		T(s)	
	Pressure	Stock Level	Pressure	Stock Level
PSO	1.53	1.66	1.00	1.10
IMC	1.74	2.05	1.19	1.66
ZN	2.35	2.82	1.60	2.32
TL	1.60	1.67	1.00	1.27

5.2 Result Analysis for Perturbed Headbox Model

5.2.1 Performance Analysis of Controller for Perturbed Headbox Model

In this section, responses of perturbed headbox model have been presented. Perturbations are added to the nominal plant because there are certain parameters which may go uncertain to certain extent. Hence, instead of a discrete plant, there is always a set (i.e. family) of plant that exist. The controller designed must be capable of handling that set of plant. In other words, the controller must perform satisfactorily for the family of plant. In this section, only performance of PSO based MIMO-PI controller have been assessed for perturbed headbox model. To begin with, step response of perturbed headbox model is shown in Figure 15 and Figure 16.

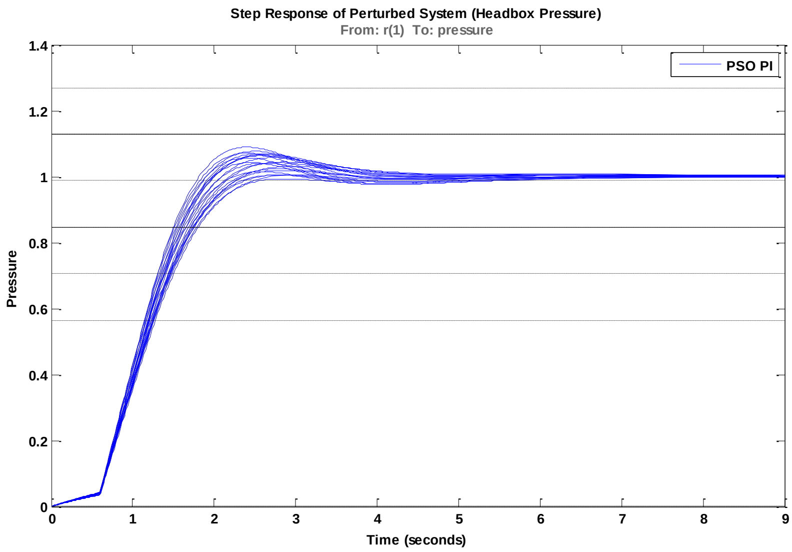

Figure 15. Step response of perturbed headbox pressure using PSO

Figure 15 and Figure 16 clearly depict that the MIMO – PI controller designed using PSO is capable of settling down the responses of set of plant (i.e. perturbed plant) with in stability limits around desired value. Similarly, load disturbance rejection has been analysed and the respective responses have been depicted in Figure 17 for headbox pressure and Figure 18 for

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stock level. From these responses, it is observed that, PSO based MIMO – PI controller is capable of handling a perturbed plant as well.

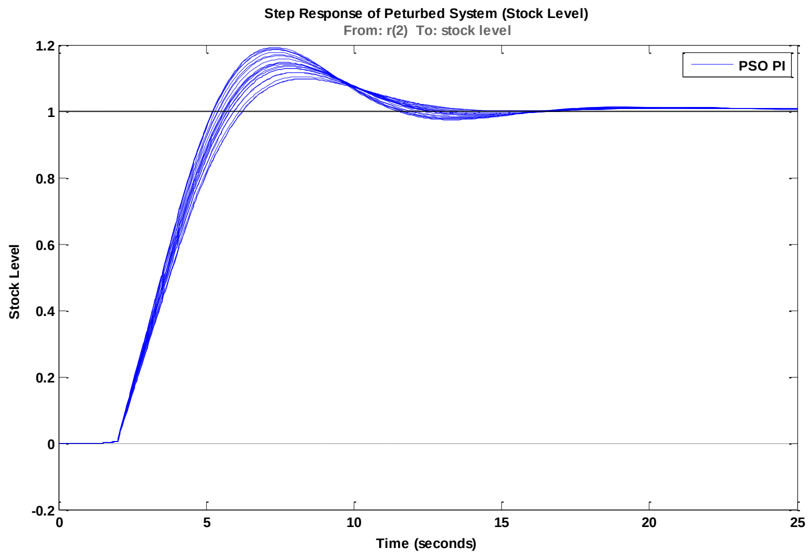

Figure 16. Step response of perturbed headbox stock level using PSO

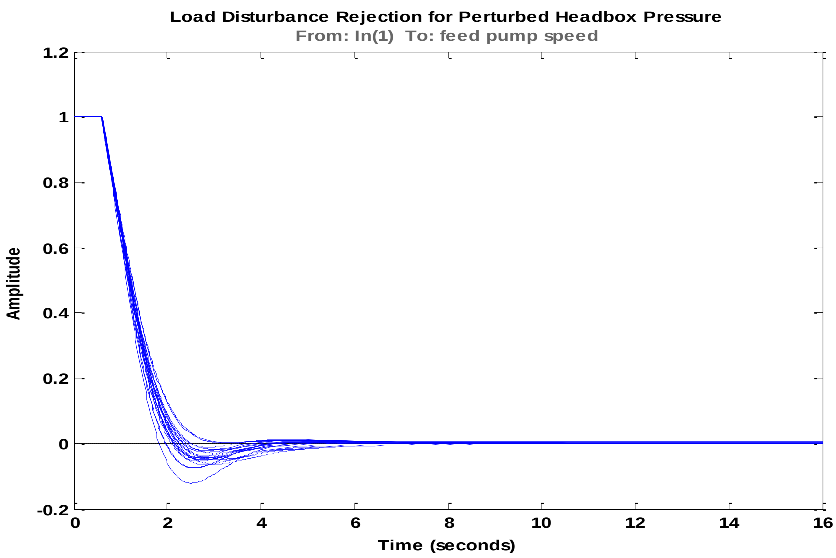

Figure 17. Load disturbance rejection of perturbed headbox pressure using PSO

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Vol. 7, Issue 2, 132-152, 2019
ISSN: 0975-1416 (Print), 2456-4281 (Online)

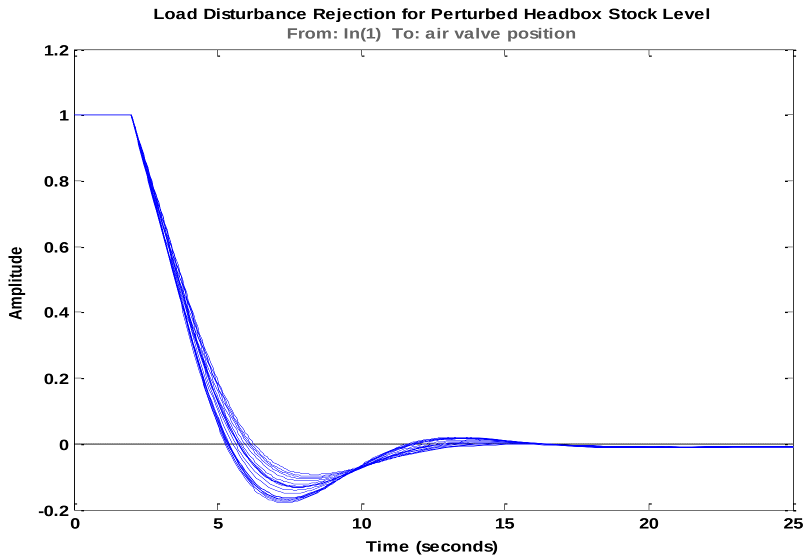

Figure 18. Load disturbance rejection of perturbed headbox stock level using PSO

5.2.2 Robustness Analysis of Controller for Perturbed Headbox Model

Worst case sensitivity analysis is carried out to check the robustness of designed controller for perturbed headbox model. Worst case sensitivity deals with robustness margins that reveals how much variation in uncertain parameters the system can tolerate. Worst case sensitivity indicates the amount of uncertainty that a control system can tolerate before violating the robustness requirement. In this section, worst case sensitivity analysis of the perturbed headbox have been presented. Also, the worst case sensitivity analysis and stability of the perturbed headbox model has been analysed using MATLAB command:

```
>> wcst = wcsens(P,C)
```

Here, “P” represents the multivariable perturbed headbox model and “C” represents the controller designed. The performance of proposed controller has been compared with the other three conventional control techniques. As mentioned earlier also that for satisfactory response, the peak value of sensitivity function shall exist between 1.5 to 2.0. The value near to 1.5 indicates the perturbed system is more stable and robust.

Figure 19 and 20 represent a comparison of nominal and worst case sensitivity analysis of sensitivity function for headbox pressure and stock level respectively with PSO based MIMO – PI controller.

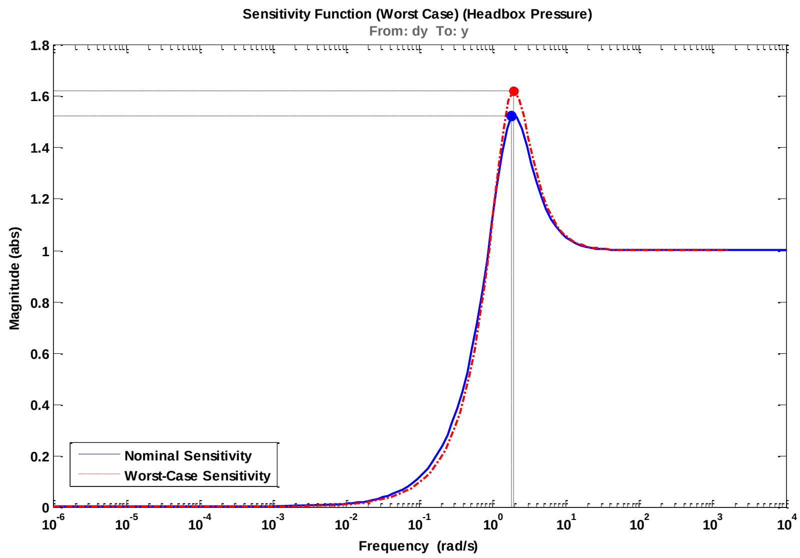

Figure 19. Sensitivity analysis of perturbed headbox pressure using PSO

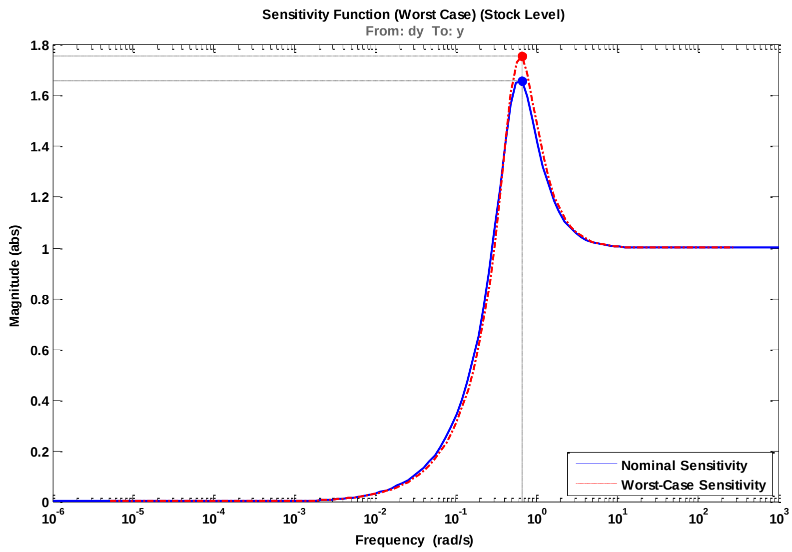

Figure 20. Sensitivity analysis of perturbed headbox stock level using PSO

Similarly, Figure 21 and 22 indicate comparison of nominal and worst case sensitivity for complementary sensitivity function.

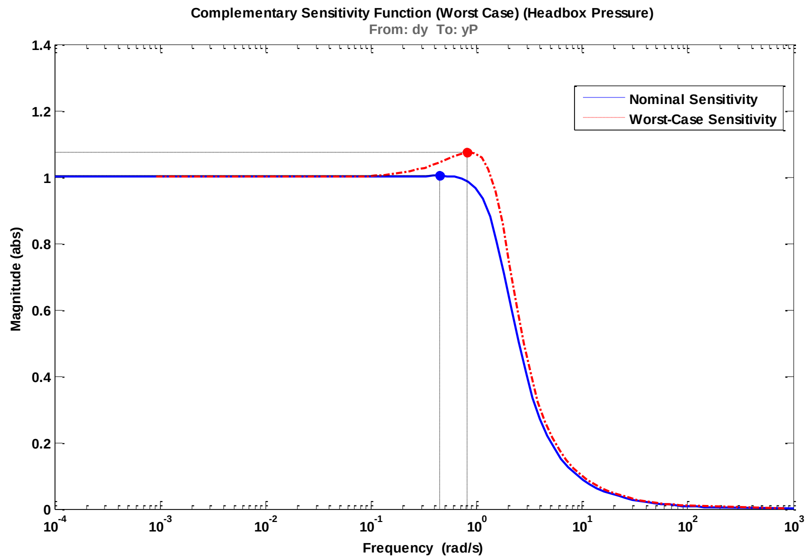

Figure 21. Complementary sensitivity analysis of perturbed headbox pressure using PSO

In all four Figures, red line (dashed) indicates worst case sensitivity plot and blue line (solid) indicates nominal sensitivity plot. Red and blue dots indicate the peak values of both sensitivities.

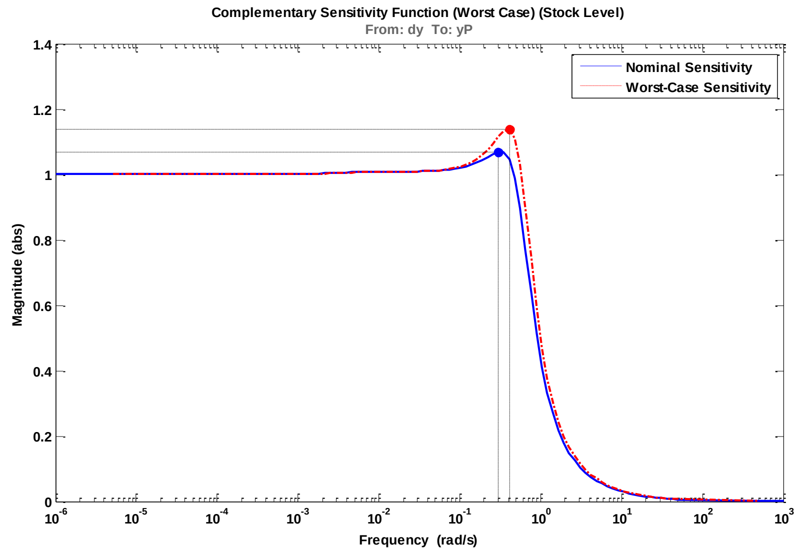

Figure 22. Complementary sensitivity analysis of perturbed headbox stock level using PSO

Table 8 presents a comparison of worst case sensitivity peak values of both sensitivity functions (i.e. S(jw) and T(jw)) for both parameters (i.e. headbox pressure and stock level) respectively.

Table 8. Worst case sensitivity analysis (peak values) of S(s) and T(s)

Controller	Worst Case Sensitivity Peak Values			
	S(s)		T(s)	
	Pressure	Stock Level	Pressure	Stock Level
PSO	1.61	1.75	1.1521	1.5162
IMC	1.75	2.03	1.6395	2.6480
ZN	2.05	2.28	2.4239	4.7785
TL	1.60	1.72	1.0476	1.6836

On comparing these values with nominal peak values as depicted in Table 4.10, it is observed that the worst case peak values are existing within the desirable range. So, it can be concluded that PSO based MIMO PI controller yields optimal robustness.

6. Conclusion

This paper has presented design and analysis of multivariable (MIMO) PI controller using heuristic technique PSO and other conventional techniques for paper machine headbox (nominal and perturbed model). On comparing the responses obtained through PSO based MIMO – PI controller, with other three conventional techniques, it has been observed that PSO based MIMO – PI controller yields optimal results in terms of performance and robustness.

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