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Technical Review on Electromagnetic Inverse Scattering

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Abstract
Electromagnetic inverse scattering is a method to identify the geometry, location and material properties of unknown target/targets using the collected scattered field data which may be due to known or unknown sources. This area of research also extended to work with the dynamic object where the objective is to retrieve velocity profile along with its general properties. Here we represent all the major development done in electromagnetic inverse scattering both in static and dynamic platform since last four decades both in time and frequency domain with its working principle, advantages, and its limitations. We also provided a comprehensive discussion of the basic theory of inverse scattering analysis both in the static and dynamic domain.

Keywords- Inverse Scattering; Microwave Imaging; Electromagnetic Forward Scattering; Terahertz Imaging; Moving Target Imaging; Radar Imaging; Non-Relativistic Imaging.

1. Introduction

Static Electromagnetic Inverse Scattering (SEIS) is an area of study to identify unknown targets with its location and shape by analyzing scattered field data generated by these targets due to known sources. Most of the case we assumed the material properties along with source properties are previously known. But with some additional workaround, this also can work unknown material properties. Practical use of SEIS is in many areas such as through the wall surveillance system, biomedical imaging, mining detection, extraterrestrial application, radar detection and ranging etc. The primary objective of inverse scattering is to construct permittivity/refractivity or conductivity profile of unknown targets. Since last four decades researcher are proposed various methods for solving SEIS problems both in time and frequency domain. Time domain inverse scattering work efficiently for the one-dimensional problem. These type of algorithms are fast but limited practical application. Whereas frequency domain inverse scattering can work in all dimension and all shape objects but solving the frequency domain problem is complicated because of nonlinearity in the problems. To overcome this nonlinearity researcher proposed many algorithms such as Born's approximation, Rytov's approximation which makes the system linear but resulting decrease in spatial resolution. Also, many optimization techniques are developed to work with nonlinearity without any approximation to get better spatial resolution.

For efficient detection, the algorithm for SEIS should address output resolution, algorithm speed, range and dimension of the unknown targets with noise handling capacity, and minimum amount of prior information requirement.

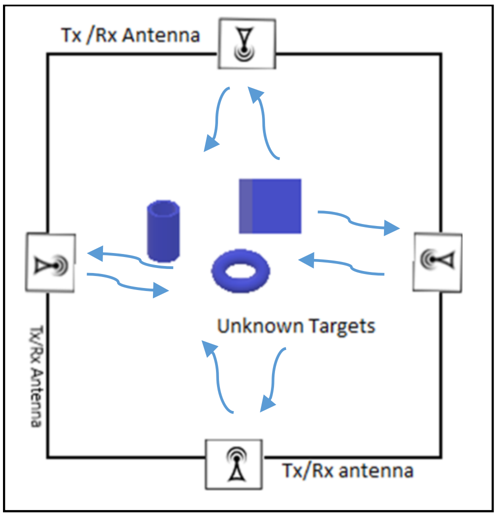

Figure 1. Electromagnetic inverse scattering problem configuration

As shown in Figure 1, generally the inverse scattering is carried out under multiple number of known sources. After the source signal gets interacted with the unknown targets the scattered or reflected field generated which are recorded in all direction for processing. Multiple transceiver helps for 360 degree coverage. As showed in Figure 1, there are four number transceiver systems to do the job.

In the case of Dynamic Electromagnetic Inverse Scattering, the objective is to trace the motion of the target by extracting the frequency shift (Doppler Shift) from the receiving scattered signal with respect to the source reference signal. There are many issues like dappled ambiguity, an abrupt change in motion, impact of different types of noise, impact of unwanted clutter and presence of additional components due to higher order scattering needed to take care. There are many time and frequency domain methods are available for velocity profile extraction. In this work, we provided a detailed discussion and comparison developed in last four decades both in static and dynamic imaging.

This paper is organized as follows. Section 2 provides theoretical background for both static and dynamic target imaging. Section 3 provides detailed discussion of the development happened in last four decades. The concluding remark is provided in section 3.

2. Theoretical Background

In this section we are presenting basic theoretical background for inverse scattering with static and moving targets respectively.

2.1 Inverse Scattering for Static Targets

The existing Electromagnetic Inverse Scattering Algorithm for static objects can be classified as following Figure 2.

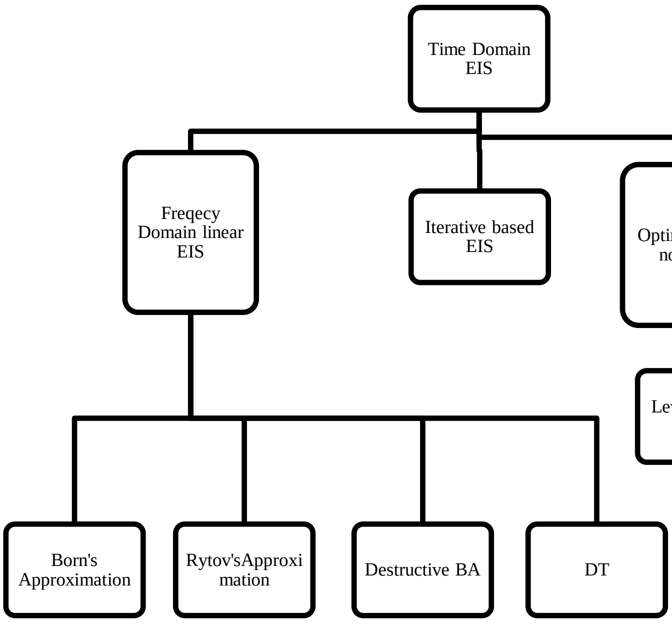

Figure 2. Electromagnetic inverse scattering algorithm classification

Inherently the electromagnetic scattering is nonlinear in nature. This nonlinearity is due to the presence of resonating structure, polarized current because of induction and also due to the presence of multiple scatterer. The general expression for nonlinear scatter field can be given in equation 1.

$$\overrightarrow{E_{Scatter}}(\vec{r}) = \int \vec{G}(\overrightarrow{r_1},\overrightarrow{r_2}).\vec{O}(\overrightarrow{r_2}).\vec{E}_{Incident}(\vec{r}).\overrightarrow{dr} + \int \vec{G}(\overrightarrow{r_1},\overrightarrow{r_2}).\vec{O}(\overrightarrow{r_2}).\vec{E}_{Scatter}(\vec{r}).\overrightarrow{dr} \quad (1)$$

Where $\overrightarrow{r_1},\overrightarrow{r_2}$ represents position vector of the observer and source $\vec{E}_{Scatter}(\vec{r})$, $\vec{E}_{Incident}(\vec{r})$ represents scattered field and Incident field respectively. $\vec{G}(\overrightarrow{r_1},\overrightarrow{r_2})$ represents Green's function of the observer with respect to corresponding source, which satisfy equation 2.

$$\nabla \times \mu_0{}^{-1}.\nabla \times \vec{G}(\overrightarrow{r_1},\overrightarrow{r_2}) - \omega^2\epsilon_0\vec{G}(\overrightarrow{r_1},\overrightarrow{r_2}) = \mu_0{}^{-1}\bar{I}(\overrightarrow{r_1} - \overrightarrow{r_2}) \tag{2}$$

Whereμ_0, ϵ_0 represents free space permeability and permittivity $\bar{I}$ is an identity matrix.

Here our aim is to retrieve the unknown target by evaluating $\vec{O}(\overrightarrow{r_2})$ which is object vector and its mathematical definition in terms of refractive index can be expressed as equation 3

$$\vec{O}(\overrightarrow{r_2}) = k^2(n(\vec{r}_2)^2 - 1) \tag{3}$$

Where $n(\vec{r}_2)$ represents complex refractive index of the unknown target and k is the wave number of incident signal.

But the above equation will have infinite number of solutions. So by the prior knowledge of the refractive index it will provide unique exact solution. There are different methods for solving the above nonlinear equation which we will discuss these types in this section.

2.1.1 Linearization Inverse Scattering Method

By Solving the above nonlinear problem is a complex task and there are many difficulties we always observed such non convergence of the equation etc. So by doing some approximation we can linearize the above equation. This can be achieved by Born and Rytov's approximation (Wang and Chew, 1989; Chew and Wang, 1990).

The Born approximation works by limiting the nonlinearity due to induced current polarization and multi scattering effect. This approximation is applicable under the condition of weak scattering and when target object is small i.e. $\vec{O}(\overrightarrow{r_2})$ is very small. With these conditions second term of right part of equation 1 can be neglected and resulting linearized version of the equation expressed as equation 4 which can be used for inverse scattering.

$$\vec{E}_{Scatter}(\vec{r}) = \int \vec{G}(\overrightarrow{r_1},\overrightarrow{r_2}) . \vec{O}(\overrightarrow{r_2}).\vec{E}_{Incident}(\vec{r}).\overrightarrow{dr} \tag{4}$$

Similarly the Rytov's approximation is applicable if the phase variation is very small and smooth then by limiting the equation 1 the resulting linearize equation with phase equivalent can be expressed as equation 5.

$$\vec{\psi}_{Scatter}(\vec{r}) = \frac{1}{\vec{\psi}_{Incident}(\vec{r})} \int \vec{G}(\overrightarrow{r_1},\overrightarrow{r_2}) . \vec{O}(\overrightarrow{r_2}).\vec{\psi}_{Incident}(\vec{r}).\overrightarrow{dr} \tag{5}$$

Where $\vec{\psi}_{Scatter}(\vec{r}), \vec{\psi}_{Incident}(\vec{r})$ represents phase of scattered and incident field respectively.

The Rytov's approximation is applicable for large scale smooth surface also under larger scattering value whereas the Born's approximation is applicable only for small scale object with small scattering value but there is no limitation on the shape of unknown targets density is calculated.

Generalized steps for Electromagnetic inverse scattering under linearized condition is presented in Table 1.

Table 1. Inverse scattering under linearized condition

Step-1: Provide the multiple number of signal from different directions towards unknown targets.
Step-2: Collect scattered field data coming from unknown targets at different directions.
Step-3: Consider magnitude for born approximation or consider phase for Rytov's approximation from the measured data.
Step-4: Evaluate Fourier transformation of above approximated measured data
Step-5: Evaluate object matrix by Inverse Fourier transformation and evaluate the refractive index profile by the help of linearized equation.

2.1.2 Optimization Based Inverse Scattering Method

This is a complete nonlinear model resulting good reconstruction because of no approximation. In this method, a simulated forward scattering was done with an initial guess of unknown targets to define a cost function or error function as equation 6.

$$\text{Cost} = \frac{\sum\left|\mathrm{E}^2_{\text{measure}} - \mathrm{E}^2_{\text{inverse}}\right|}{\sum\left|\mathrm{E}^2_{\text{inverse}}\right|} \tag{6}$$

Where $E_{\text{measure}}, E_{\text{inverse}}$ is measured Electric field in laboratory and simulated Electric field during inversion. The objective to use a different technique such as level set algorithm (Eskandari and Safian, 2010) to minimize the cost function by changing the initial guess. After some iteration reconstructed object can be noted with very less error. This method also works efficiently in the presence of noise with respect to linearize Inverse Fourier transform method because of inbuilt noise handling capacity. Generalized steps for optimization based Electromagnetic inverse scattering presented in Table 2.

Table 2. Optimization based electromagnetic inverse scattering

Step-1: Provide the multiple electromagnetic signal from a different direction towards unknown targets.
Step-2: Collect scatter field data coming from unknown targets at different directions.
Step-3: Simulate the forward scattering using initial guess by any computational methods such as FDTD, MOM, and FEM.
Step-4: Find out Cost function using equation 6
Step-5: Use inverse scattering optimization technique to find out next required change in last guess to minimize the cost function value.
Step-6: Repeat the 3, 4, 5 steps until desire convergence is achieved to get the final target.

2.1.3 Time Domain Inverse Scattering Method

For time domain inverse scattering analysis the fundamental time-domain integral equation as in equation 7 is utilized to obtain simple recurrence formula.

$$E_{total}(z,t) = E_{incident}(z,t) - \frac{\eta_0}{2}\int_0^d \sigma(z')\,E(z',t')dz' - \frac{\eta_0\varepsilon_0}{2}\int_0^d [\varepsilon_r(z') - 1]\frac{\partial}{\partial x}E(z',t')dz' \quad (7)$$

Where t' is retarded time, t is observation time, z' is position vector scatter and z is position vector of observer η_0, ε_0. Represents free space refractive index and free space permittivity. ε_r is relative permittivity of the scatterer $E_{incident}$ is incident electric field and E_{total} is total electric field. Generalized steps for time domain electromagnetic inverse scattering presented in Table 3.

Table 3. Time domain electromagnetic inverse scattering

Step-1: Provide the multiple electromagnetic signal from a different direction towards unknown targets.
Step-2: Collect scatter field data coming from unknown targets at different directions.
Step-3: Simulate the forward scattering using initial guess by any computational methods such as FDTD, MOM, and FEM to compute the field matrix.
Step-4: Solve for refractive index profile using equation 7.
Step-5: Find out the error with new updated profile with measured field and choose some intermediate refractive index.
Step-6: Repeat the 3, 4, 5 steps until desire convergence is achieved to get the final target.

2.2 Inverse Scattering for Dynamic Targets

For imaging of dynamic targets we need to perform some additional steps along with static imaging to retrieve the profile of the target object. So the static imaging is a time instance imaging in temporal space. Depending on the methodology the dynamic imaging can be carried out using following types. The classification of dynamic imaging as shown in Figure 3.

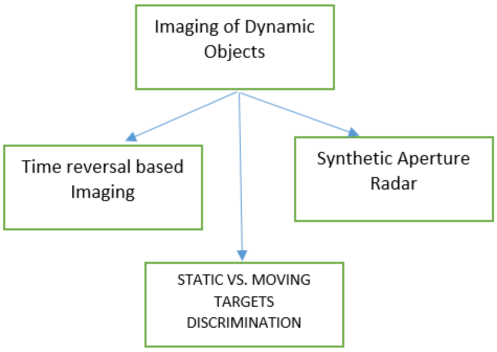

Figure 3. Dynamic imaging technique classification

2.2.1 Doppler Shift Base Imaging

Doppler shift base imaging also known as radar base imaging. For this imaging algorithm to work should be a relative velocity between trans-receiver system and target.

The basic radar equation can be written as

$$\eta(t,v) = \int \rho(t',v')\,\chi\left(t-t',v-v'e^{\frac{i\left((t-t').(v-v')\right)}{2}}dt'dv'\right) + Corellation\ noise \qquad (8)$$

Where ρ is signal strength scale factor which is also known as object function t , v represents time range and velocity of the target object .The correlation noise is due to the scatter field present due to unwanted clutter and χ is radar ambiguity function.

For computational based imaging the objective is to determine the value of object function from the received data set which is a nonlinear problem. But there are many linear methods are available which we will consider further. As shown in the Figure 2 the reconstruction can carried out with the spatial aspect, temporal aspect, and spectral aspect. As shown in the Figure 4 by linearizing we can subdivide the radar-based imaging into three subgroups as Spectral-Temporal based Imaging, Spatial-Temporal base imaging and Spectral-Spatial base imaging. We will further discuss the basic working principle for all of these.

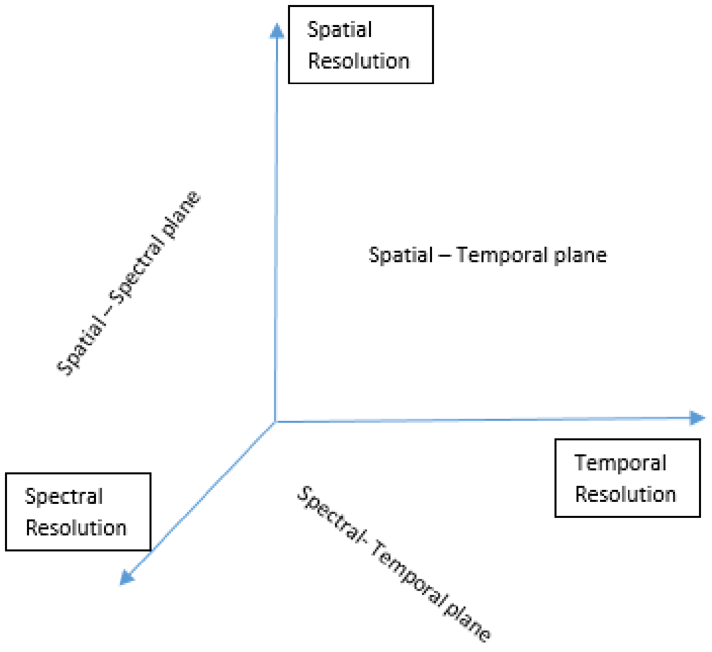

Figure 4. Doppler shift base radar imaging domain

2.2.1.1 (Spectral-Temporal Imaging) Doppler only Imaging

This is a high Doppler resolution technique where a fixed frequency waveform is exited from the antenna array which is on a moving platform. The Doppler shift will be the superposition of all returns due to relative moving scatterer due to same relative velocity. This result creates a hyperbola which is also known as Iso-Doppler hyperbola curve or isodop. The objective is to reconstruct the target from isodop. This process gives superior spatial resolution to the unknown target but at the cost of temporal information. Generalized steps for Doppler only imaging presented in Table 4.

Table 4. Doppler only imaging

Step-1: Provide the multiple electromagnetic signal from a different direction towards unknown targets.
Step-2: Collect Scatter field data coming from unknown targets at different directions
Step-3: Simulate the Forward Scattering using initial guess by any computational methods such as FDTD, MOM, and FEM to compute the field matrix.
Step-4: Solve for refractive index profile using equation 7.
Step-5: find out the error with new updated profile with measured field and choose some intermediate refractive index.
Step-6: Repeat the 3, 4, 5 steps until desire convergence is achieved to get the final target.

2.2.1.2 Moving Target Indicator (MTI) and Pulse Doppler (PD) Radar

MTI bases radar uses the phase measurement to identify the velocity of unknown target/targets. Generalized steps for moving target indicator imaging presented in Table 5.

Table 5. Moving target indicator (MTI) radar

Step-1: A transceiver antenna generates a pulse source signal and also collect the reflected signal which may be due to unknown target/targets.
Step-2: A phase comparator provided with transmitted wave as reference and the received reflected wave to calculate the phase shift and for a complete cycle phase change represents half of the wavelength as change in range of the target.
Step-3: A pulse comparator circuit use to discriminate between moving and stationary target.
Step-4: Clutter are removed using cancellation circuit where it eliminates non-zero phase average.

Whereas the Pulse Doppler radar uses the frequency measurement from the line of sight for velocity detection of unknown moving targets so instead of phase detection. Generalized steps for Pulse Doppler radar imaging in Table 6.

Table 6. Pulse Doppler (PD) radar

Step-1: A transceiver antenna generates a pulse source signal and also collect the reflected signal which may be due to unknown target/targets.
Step-2: A mixture is use to find out the frequency spectral difference between reference and received signal.
Step-3: A Doppler filter is used select the signal which are only due to movable targets.
Step-4: The signal from Doppler filter output then further post processed and feeded to display unit.

2.2.1.3. Synthetic Aperture Radar for Moving Object

Here the spatial resolution of the target's image depends on the beam width of the antenna and which depends on the antenna aperture. But it was very difficult to handle with a very large antenna in a movable system. To overcome this Carl Wiley in 1954 purposed a different method to synthetically increase the aperture of the antenna without increasing the physical antenna size which is otherwise known as Synthetic Aperture Radar (SAR) (Borden and Cheney, 2005; Stuff et al., 2004; Zheng et al., 2015).

Let's assume that the antenna system is attached to an airborne system which is moving with velocity "V" and generating a signal of frequency "F". When the signal gets reflected by any static object then due to the Doppler Effect the new shifted frequency will be.

$$F_{reflected} = \frac{2*V}{C} F \ \cos \emptyset \tag{9}$$

Where $F_{reflected}$ is reflected signal, C is velocity of light and $\emptyset$ is angle of inclination of the target.

Similarly the Doppler Shift difference between two different static targets with $\emptyset_1$,$\emptyset_2$ inclination angle respectively can be given as

$$F_{reflected\ 1} - F_{reflected\ 2} = \frac{2*V}{C} F \ \sin \emptyset \ (\emptyset_1 \ - \ \emptyset_2) \tag{10}$$

Where $F_{reflected\ 1}$,$F_{reflected\ 2}$ are frequency reflected signal from two very close static target. Generalized Steps for SAR radar imaging presented in Table 7.

Table 7. Synthetic aperture radar

Step-1: After getting the reflected data in the receiving antenna array, it was quantized and stored in digital grid.
Step-2: 3D Fourier transform is used to convert the above digital data in frequency domain.
Step-3: Then the data with maximum magnitude represented in the 3D image platform.
Step-4: From signal history a further scenario was synthesised.
Step-5: Least square fitting technique is used to determine best fitted sample among all collected sample by the help of synthetic data set.
Step-6: Residual synthetic signal was generated and stored as signal history for use in further steps.
Step-7: all the above steps are repeated until desire objective is achieved.

2.2.2 Static Vs. Moving Targets Discrimination (Fouda, 2013)

In this process, the experimental platform should contain some stationary targets along with some slow-moving targets. Initially, the algorithm tries to detect stationary targets using any time reversal algorithm. And uses this result as prior knowledge for the next part of the problem to find the solution for a moving object. Generalized steps for discrimination based algorithm imaging presented in Table 8.

Table 8. Static vs. moving targets discrimination

- Step-1: Electronically recording the multi static data matrix (MDM) for each transmitter.
- Step-2: Algorithm for stationary target
    - Step A: Apply the decomposition of the time reversal operator (DORT) on average MDM and further find the value of eigenvalues and eigenvectors.
    - Step B: Find out the locations and material characterization of stationary targets using calculated eigenvalues and eigenvectors.
    - Step C: Simulate the forward scattering using green function for the detected stationary objects.
- Step-3: Algorithm for moving target.
    - Step A: Apply The DORT on differential MDM and further find the value of eigenvalues and eigenvectors.
    - Step B: Find out the number and material characterization of moving targets using calculated eigenvalues.
    - Step C: Find out the location and geometry of moving targets using calculated eigenvalues and the data collected from “C” step of stationary target evaluation algorithm.

2.2.3. Time Reversal Algorithm

The primary constraint of this algorithm that computation time should be less than minimum tracking time of the moving target. This algorithm works where scattering due to target is very high with respect to scattering from ambient clutter. Generalized Steps for Time Reversal Algorithm Imaging presented in Table 9.

Table 9. Time reversal algorithm

- Step-1: Use any standard static target algorithm to find the initial position of the moving target.
- Step-2: Using beam steering method for antenna array try to obtain proper focus for the detected target.
- Step-3: For detecting the target’s the normalized phase conjugated scattering vector is projected onto the normalized steering vectors of a synthesized imaging domain.
- Step-4 : In the projection surface (image surface) where the maximum projection occur due to constructive addition are considered as new target location.
- Step-5 : Step 3 and 4 repeated for desired time period for tracing the moving target.

3. Discussion

This section deals with the review of various research papers right from 1970 regarding all major developments in the field of EIS. Proceeding from the discovery of Maxwell equation researchers use the field matter interaction and boundary value problems in different sectors including scattering analysis further by the use of Green’s function modelling of scattering problems become easier. The researcher also used this analysis to reconstruct the geometry as discussed earlier. In 1975, a mathematical foundation was developed to linearize the scattering problem using the Born and Rytov’s approximation to perform a direct inverse scattering (Iwata and Nagata, 1975). Further a one-dimensional time domain EIS was developed which suffer difficulties for implementing in higher dimension (Lesselier, 1978). By further extending the work of Rytov’s approximation a back propagation filtered was developed which shows more efficient output which is also known as Diffraction Tomography. To overcome the limitation of Born’s approximation further many numbers of modification are done to work under high scattering environment such as the introduction of the Relative Residual Error (RRE) (Chew and Wang, 1990). Keeping military application in mind a faster algorithm was developed known as the Antenna Synthetic Aperture Radar Imaging-ASAR which is also working on the mobile platform and able to recover the 3D image of the target (Ozdemir et al., 1998). To receive pinpoint accuracy and high resolution of unknown target linear sampling method is used. This method requires a huge amount resource and data for computation. Similarly to work with nonlinear EIS platform another recursive function approach where the desire unknown target profile can be achieved using a cost function. (Cakoni and Colton, 2003). These optimization approaches are highly efficient for EIS problems under the presence of noise. A detailed review of static EIS algorithms is presented in Table 10.

Similarly, there are many developments happens for dynamic objects detection in the last five decades. In 1967, W. Brown provided the basic mathematical background for Synthetic aperture radar and proposed the use of Fourier transformation mechanism to reconstruct the dynamic target (Brown, 1967). In 1980, J. L. Walker developed the mathematical background for imaging rotating target (Walker, 1980). Further, H. E. Rowe provides the method for one-dimensional radio imaging using radio temperature for tracing a point which was moving with constant velocity. In 1991, M. Soumekh presented the algorithm to detect the dynamic object using Bi-Static SAR. X. L. Xu provides the method to use Limited- diffraction beams to image biological tissue and for other applications using Bessel beam and Doppler shift. In 1997 B. Friedlander provided Velocity SAR method to reconstruct a 3D image with high spatial resolution. Mark S. Roulston purposed a Doppler only method to reconstruct the image of the polar region in different planets. B. Zheng in 2000 provided the method for imaging for fast maneuvering targets (Zheng et al., 2000). In 2007, M. I. Pettersson provided a method to image the moving target in four-dimensional discretized space. In 2013 Matteo Pastorino presented a computational platform to image and trace axially moving cylinders (Fouda, 2013). In 2015 A. Zhuravlev presented a combined method to use Multi-Static radar and Video Tracker to image moving targets (Pastorino et al., 2015). In 2017, Q. Yaolong provided the use of compression sensing technique and snapshot imaging radar to track moving target (Yaolong et al., 2017). A detailed review of moving EIS algorithms is presented in Table 11.

Table 10. Comparative analysis for static object imaging

S. No.	Paper Title	Proposed Algorithm	Result	Remark
1	Calculation of refractive index distribution from interferograms using the Born and Rytov's Approximation (Iwata and Nagata, 1975).	Author Introduced methods for using the Born and Rytov's approximation for the purpose of electromagnetic inverse scattering with approximated linearize field .Here inversion is done using inverse fourier transformation.	Here the derivation was done for a homogenous two dimensional cylinders with approximated linearize equation. It is observed that Born's approximation is good for small scattering with small unknown targets whereas Rytov's approximation is good for smooth large objects but no limitation as Born's approximation.	This paper mainly focused on developing linearize two dimensional inverse scattering platform in the frequency domain which can further develop to work with higher dimension.

2	Determination of index profiles by time domain reflectometry (Lesselier, D. 1978).	An exact time-domain Integral equation is utilized to obtain simple recurrence formula. Further it is used to identify the field values across all discrete space time position. Which further utilize to linearly obtain unknown profile. Truncated data have been generally smoothed by “gravity centre method”.	Here three different one dimensional unknown profiles are used for detection 1. Step Homogeneous profile 2. Continuous linear profile 3. Sine-Square buried inhomogeneous profile. The reconstruction was done within 3 to 5 numbers of time steps movement for a 10* wavelength dimension (approx.) targets.	This algorithm is a time reversal approach which has noise handling capacity. This can be useful for source determination also. This approach is good for one dimensional problem but not for higher.
3.	Iterative determination of permittivity and conductivity profiles of a dielectric. Slab in the time domain (Tijhuis, 1981).	This is a recursive algorithm where forward scattering simulation is done in every time step which helps to select points on unknown target whose electric field strength shows accurate value over space time discrete points.	In this presented paper, the author tested the algorithm with five different one dimensional conductivity profile with very smooth variation and the result observed second decimal accuracy after five number of iteration	Here author provides a novel recursive approach to solving inverse scattering. This algorithm works perfectly for simple smooth targets but it is inefficient for complex random target.
4.	A Computer Simulation Study of Diffraction Tomography (Devaney, 1983).	Here the author developed an Inverse scattering algorithm by modelling interaction between field and target using Rytov’s approximation and subsequently devolved filtered back propagation algorithm to transfer the model into image space using Green’s function.	By the help of a digital computer the algorithm was implemented using 129*129 pixel based system to retrieve an unknown target which consists of four cylinder (2D) with dimension of five times of wavelength was implemented. The imaging was achieved by 7th iteration.	Here the author moved to ultrasonic range from traditional x-ray range and it work efficiently. This algorithm is valid only where Rytov’s approximation is valid.
5.	An iterative solution of two-dimensional electromagnetic inverse scattering problem (Wang and Chew, 1989).	The author introduced a nonlinear inverse scattering algorithm. It is a two dimensional recursive inverse scattering algorithm without the Born and the Rytov’s approximations limitation. The algorithm solves the problem with the Born approximation followed by forward scattering problem was solved using the Method of Moment and further the reverse problem was solved by modified Newton method which removes the limitation. These processes will be repeated until desire objective achieved.	Here the algorithm was subjected to multiple range of frequencies starting from 10 MHz to 100 MHz. Different types of unknown profiles are used such as Smooth varying permittivity distribution, Discontinuous permittivity distribution, Sin-like permittivity distribution, Axially asymmetric permittivity distribution .All the reconstruction was achieved by 6th iteration.	This algorithm works efficiently also when Born And the Rytov’s approximations are breaks. This method has a robust noise handling capacity. This algorithm can also implemented under the framework of diffraction tomography.
6	Reconstruction of Two-dimensional permittivity distribution using the Distorted Born Iterative method (Chew and Wang, 1990).	Here Born approximation is used to linearize the problem and Method of Moment (MOM) is used for forward scattering for use in recursive platform. Here Relative Residual Error (RRE) is calculated by comparing the MOM result with customized green function output. Here RRE will help to achieve optimization.	The author used 100MHz signal as source with a background of 25 dB Signal-to-noise ratio. Sin-like permittivity distribution was used for reconstruction which is tested with a noisy environment and noise free environment. The convergent of the solution achieved after 15 iterations.	As compared to Born iterative method the Distorted Born Iterative method is having a faster convergence rate. This method can be extended to solve three dimensional problems also.

7	A modified gradient method for two dimensional problems in tomography (Kleinman and Van Den Berg, 1992).	Here a relaxation technique is used in forward scattering problem to minimize the RRE to achieve better convergence. Here the cost function is scalar. The update of the initial guess in each iteration is achieved by using the conjugate gradient method.	There are three separate profiles are taken as unknown targets, such as Gaussian surface, multi cylinder, discontinuous cubical structure. With the above structures the convergence achieved by 64, 64, 512 number of iteration respectively.	This algorithm provides larger operating range of scattering field amplitude with respect to Born's approximation. Also it has a faster convergence rate.
8	A contrast source inversion method (Abubaker and Van Den Berg, 2001).	The proposed algorithm is a special case of Source-Type Integral Equation (STIE) method which is useful to map measured field data with source distribution over scatterer. Here the author introduced a special error function which have additional component presenting error in the form of the state equations along with normal error in the data equations	There are three separate profiles are taken for imaging such as a Gaussian surface, Two cylinder (2D),discontinuous cubical structure whose images are retrieved by 64,64,512 number of iteration respectively. The result is far better with respect to modified gradient method.	The author presented a simple but versatile platform which can accommodate different types of source and unknown targets.
9	Antenna Synthetic Aperture Radar Imaging-ASAR (Ozdemir et al., 1998).	In Synthetic Aperture Radar, the high resolution image is formed by processing multiple radar images which are from antenna radiation data and also to pinpoint the target. This platform uses inverse Fourier transformation platform for a multi-frequency, multi-aspect far-field data which are collected from antennas mounted on many platforms	Here the two dimensional projected ASAR image was formed using the data collected from the antenna connection on the nose of a fighter jet. Different side views are collected by this platform and which are further used to form the resulting image was formed in 2D and 3D. Here the experiment was made for a jet target.	This developed is very helpful for getting pinpoint the location of the target. It has a wide verity of application starting from military to scientific work. This development provides mobile platform to work along with faster detection capacity.
10	The linear sampling method for cracks (Cakoni and Colton, 2003)	This method provides a special platform to represent the scatterer by the help of solution of a linear integral equation. It uses far field data for reconstruction for higher linearity. Here the author developed the mathematical background to detect any crack in a homogeneous conducting cylinder.	Here across a unit circle 32 sources and observation points equally distributed. And TM-polarized scattered far field data were recorded with 5% noise. The unknown target is the cylinder is a perfect conductor having a line and a curve crack. After inversion the algorithm able to give accurate crack details.	Here Detection is very accurate. Also in the presence of noise. Disadvantages of this method is that it needed considerable amount of field data and computational resource. And it also doesn't work efficiently for non-homogenous target.
11	An Inverse Scattering Method Based on Contour Deformations by Means of a Level Set Method Using Frequency Hopping Technique (Ferrayé, 2003).	The author provides a method to reconstruct the unknown target using level set algorithm which provides a dynamic deformation platform which operates under a cost function. The cost function is the difference between the measured field and forward simulated field.	Here highly precise reconstruction achieved for following test profiles. Rocket-shaped object with 242 iteration, no convex smooth-shaped object with noise-corrupted data from 84 iteration and of three objects simultaneously with noise by 175 iteration.	This algorithm provides highly accurate reconstructions in relatively short computational times.

12	Investigating the enhancement of three dimensional diffraction tomography by using multiple illumination planes (Vouldist et al., 2005)	Here the author introduces the extension of Direct Fourier Interpolation (DFI) and Filtered Back Propagation (FBP) algorithm for 3D inverse scattering and also all mathematical requirement for the same. The algorithm retrieves the projections of unknown target in different 2D planes and reconstruct the 3D image from the projections.	Here the algorithm was tested with the following profiles, such as homogeneous cone, stepped cylinder, simple cylinder.	This algorithm able to construct of a uniform varying object with one plane data. But for more complex target and for, more accurate data from multiple planes are needed.
13	A direct sampling method to an inverse medium scattering problem(Ito et al., 2012)	This algorithm provides a tool to obtain the shape of unknown homogeneous target with very limited no of incident directions such as one or two number of sources. This algorithm is strictly direct and doesn't depend on matrix inversion as it computes the inner product of the scattered field with fundamental located at needed sampling points.	Here the author tested the algorithm with one square profile, two square profile, two cubic profile, and ring-shaped profile. This imaging is done by collecting data over 600 number of recovers which are distributed uniformly over a cube.	This algorithm has a good noise handling capacity. With limited number of excitation it exhibit good reconstruction of the unknown targets.
14	Inverse Scattering Using Scattered Field Pattern(Linkoon P. Meenaketan et al., 2016)	The proposed algorithm can retrieve the object geometry by the help of scattered field pattern due to electromagnetic plane waves. Both TM and TE polarization are considered for better numerical accuracy. This algorithm is inspired by the level set method and the author used field curvature value as cost function for inverting using level set algorithm.	For the testing of this algorithm four different profiles are used such as a rectangular profile, triangular profile, pentagon profile, and a rocket shape profile. The computation is done with 500 * 500 grid points. The platform was having four numbers of plane wave excitation existing in all four direction. And hundred numbers of recovers are there in each direction. Construction was achieved by 37,35,40,42 numbers of iteration respectively.	This algorithm provides a faster platform for inverse scattering. For simple targets it is able to perform efficient imaging but not well for complex targets. Father development of this algorithm can be made to detect 3D objects and dynamic objects.

Table 11. Comparative analysis for relatively moving object imaging

S. No.	Paper Title	Proposed Technique/Algorithm	Result	Remark
1	Synthetic Aperture Radar (Brown, 1967).	In this work, the author provides needed the mathematical platform for a side-looking synthetic aperture radar and introduces different constraints like radar ambiguity and impact of phase error. The author also provides methods to improve intrinsic resolution which further utilized for optimization process.	The mathematical platform for imaging of a rotating target field was developed also provides the technique to find out the average resolution for unknown target.	The author provided basic idea and mathematical platform for utilizing pulse base radar for the purpose of imaging.
2	Range Doppler imaging of rotating objects (Walker, 1980)	The author purposed a technique to store the return pulse from a rotational object in an angular coordinate system (polar format film storing) so that smearing effect can be compensated. The resulting data set further represented in the 3D Fourier region so that needed image can	The author purposed a technique to store the return pulse from a rotational object in an angular coordinate system (polar format film storing) so that smearing effect can be compensated. The resulting	This work was able to extract the 2D and 3D image of the target with its velocity profile. The polar storing technique provides a more resolute image with respect to a Cartesian coordinate system. The author also

		be obtained by taking the inverse Fourier transform.	data set further represented in the 3D Fourier region so that needed image can be obtained by taking the inverse Fourier transform.	took care the conjugate image ambiguity problem here.
3	Synthetic radar maps of polar region with a Doppler only method (Roulston and Muhleman, 1997).	The author introduces a technique to image the polar region of any planet by the help of only Doppler shift information and radon transformation. Here the forward scattering was simulated computationally. Further, the inversion was done using Nievergelt inversion technique. This method able to handle noise due to Spacecraft orientation drift, Orbital altitude drift, Thermal noise and quantization noise.	The algorithm tested with 900KM^2 simulated test area and the reconstruction is possible for 1KM2 where imaging was done above 150 ± 5 KM altitude with 10W radar power and 1000K noise temperature.	The given technique able to form a decent resolute image in the presence of high noise temperature and with less computational arrangement which makes it very fast system.
4	Fast Back projection Algorithm for Synthetic Aperture Radar (Yegulalp, 1999).	In this work, the author introduces very fast back propagation algorithm for SAR imaging. Here total Synthetic aperture divided into multiple numbers of sub-aperture and use standard back propagation algorithm to image individual sub-aperture. The final result is the sum of all sub results.	Here the algorithm was tested with 20Mhz to 90Mhz with 1.5m*1.0m pixel size with 2000*2000 pixel resolution from a altitude of 4KM .The imaging was performed for a relative moving cylinder with different scaling factor. The imaging was possible with 30 time faster than standard back propagation algorithm.	This development provide very faster algorithm for reconstruction in 2D platform. Also work in presence of noise. The speed gradually reduces with increase in scaling factor. So this tradeoff between computation speed and scaling can be customizable depending on requirement.
5	Principles and algorithms for inverse synthetic aperture radar imaging of maneuvering targets (Zheng et al., 2000).	The author presented a radar imaging algorithm where the target is having small maneuverers (non-cooperative) using first order polynomial approximation. This dynamic imaging was done in Temporal-Spectral plane.	Here the target is a Yak-42 fighter jet whose dimension is 36.38m*34.88m*9.83m .The radar working in C band and with band width of 400GHz. The radar platform is at 5 km altitude and imaging is done for a target with distance of 33.5 KM. The result is obtained in Time-Frequency distribution and Crosse range imaging was achieved at different time instance.	This technique is very useful to detect small maneuver of the target. It takes very less computational time and able to trace the object movement.
6	Imaging moving objects in 3D from aperture synthetic aperture radar (Stuff et al., 2004).	This algorithm takes 1D data receiving data as input which is due to a moving object and forms a 3D image. This also handle the smeared image which is due to moving target by taking history data as reference.	This algorithm was implemented both for 2D and 3D imaging. For 2D imaging it able to create 600*280 resolution and for 3D image it creates 90*128*160 vox image.	As it take the help of history data it requires high data store and high computational memory and time but provides a better imaging. It also works for non-cooperative targets.
7	Electromagnetic inverse scattering of axially moving cylindrical targets (Pastorino et al., 2015)	This algorithm uses 8 different modes od TM polarized excitation is used as source. After receiving the scattered field the algorithm truncated all 8 modes data for further analysis. Swarn based optimization technique is used for optimization in inverse scattering purpose.	For testing the algorithm a moving cylinder od 0.45 m outer radius and 0.40 inner radius is the source signal exited at 400MHz. The cylinder was moving with uniform axial velocity. The reconstructed image was presented with 10 different time instant (position profile).	The presented work only with initial momentum approximation. This algorithm able to extract both permittivity and velocity profile of the moving cylinder. This development specifically done for multi-disciplinary application.

8	Electromagnetic Time-Reversal Imaging and Tracking Techniques for Inverse Scattering and Wireless Communications (Fouda, 2013).	The author provided two different approach 1. Non differential approach: Where the idea is to first locate the moving target using any static imaging algorithm further proper focusing achieve using beam steering method and further using this information as initial value to trace the movement of moving object by comparing it with new updated data. In the second method (differential method two different simultaneous signal is use as excitation with targeting two separate points which are very close to actual target) and at receiving point the algorithm works with difference two signal.	The Algorithm was tested with simulated environment with cylindrical pillar target while moving in homogeneous background and moving under discrete clutter. This algorithm able to trace the movement along with locating multiple numbers of clutter.	This algorithm able to work in presence of noise and clutter but not able to simultaneously generating permittivity profile. A separate setup needed for spatial imaging. The temporal resolution can easily achievable by controlling the signal and processing time.
9	Snapshot Imaging Radar for Moving Target Detection Based on Distributed Compression Sensing (Yaolong et al., 2017).	This algorithm reconstruct image under spatial – temporal resolution plane. A combination of snapshot radar and distributed compression sensing technique developed to handle sparse data due to moving target .The temporal frame rate gives temporal resolution where the spatial resolution handled by the help of bandwidth of central frequency of source signal.	A pendulum system is used to test under tis algorithm .190 numbers of azimuths are used. The time resolution was 4usecond .This process able to form 20 frame /6.5 second for this setup and also able to reconstruct the profile by using 20% of echo data.	A correlation technique is used to find the amplitude and phase difference between transmitted and reflected signal further which was used to determine the velocity profile and 2D spatial resolution. This imaging platform provide high resolute data in presence of noise as noise filtered out during convolution process

4. Conclusion

Due to the availability of digital computation platform the field of Electromagnetic Inverse Scattering (EIS) developed much rapidly since last three decades with many potential applications at the same time, there is wide range of scopes for the future for EIS because of the huge development in the field of digital computing. EIS is used in different application such as biomedical imaging, radar detection, crack detection, ground penetrating radar, through wall radar etc. due to its efficiency and accuracy. Due to the development of different optimization techniques in the last decade further refinement of EIS is also possible.

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